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Cost Analysis of Two-Similar-Unit Cold Standby System With three States Under Human Failure Using First Order Linear Differential Equations

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ABSTRACT

Many authors have studied the two-similar unit redundant system with three states to evaluate various reliability parameters, but no attention was paid to the reliability evaluation due to human failure. Others have studied the two-dissimilar unit redundant system with three states to evaluate various reliability parameters, but they did not consider the cost function. This study determine the effect of human failures on the reliability of the system and determine the cost function. In this paper the MTTF, availability and cost analysis of a two-similar-unit cold standby system under human failure is discussed. The proposed system has been investigated under the assumption that each unit works in three different states: normal, partial failed ad total failure. The system suffer two types of failures: viz; unit failure and human failure. Failure and repair time distributions are exponential. Using first order linear differential equations the system characteristics have been obtained. Also the cost function of the system is obtained. A special case for the proposed system was given in which human error is not considered. The results indicated that the system without human failure is more greater than the system with human failure with respect to the MTSF, steady state availability and the profit. Conclusion: These results indicated that the system characteristics without human error are more greater than the system with human failure. We concluded that the system without human failure is better than the system with human failure with respect to the MTSF, steady state availability and the profit.

Keywords: Reliability , Cost analysis, Mean Time to System Failure (MTSF), steady-state availability, busy period, profit function, linear first order differential equations.

1- Introduction

In various reliability systems often come to maximize the profit. The profit of the system depends upon the cost incurred. The standby unit support increases the reliability of the system. On the failure of the operating unit, a standby unit is switched on by switching device. As human beings become involved in the systems, their abilities and limitation are manifested in their performance of mission tasks. Since human are essential to the operation of such systems, it is important to measure the effect of human performance on system reliability. Human error is defined as a failure to perform a prescribed job which could result in damage to equipment and disruption of scheduled operation. Human errors can be categorized as follows: maintenance error, design error, operational error, inspection error, or handling error.

Many authors have studied the two-similar unit redundant system with three states to evaluate various reliability parameters, but no attention was paid to the reliability evaluation due to human failure (Lasanovsky, 1982), (Hatogama Y, 1997), (Dhillon B. S, 1980). (Gupta P. P. and Rakesh Kumar Sharma, 1986) consider the MTTF analysis of a two-unit repairable redundant system with two states under human failure, but they don't consider cost function. (S.S. Elias and A.S. Hassan, 1989), (Goel L.R. et al, 1985) investigate the cost analysis of some systems. Also, some researchers evaluated various reliability parameters of three states systems, but they don't consider the human failure and cost function (El-Saied KH, M Soleha A.B,).

(Gupta R. and Mittle M 2006) have studied stochastic analysis of a compound redundant system involving Human Failure. (M.Y. Haggag 2009) has studied Cost Analysis of Two-Dissimilar-Unit Cold Standby System with three States and Preventive Maintenance using Linear First Order Differential Equations. (M.Y. Haggag and Ahmed Khayar 2011) have studied Profit Analysis of Two-Dissimilar-Unit Cold Standby System With Three States under Human Failure.

The purpose of this paper is to study the MTTF, Steady-state availability and cost analysis of a two-similar-unit repairable redundant system with three states under human failures. We analysis the system by using first order linear differential equations. A case study of the effect of human error on the system performance is shown theoretically and graphically.

2- Assumptions: The following assumptions were adhered to in this work:

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1. The system consists of two-similar units, one is main and the other is its standby
2. Initially one unit is operative and the other unit is kept as cold standby.
3. A switch-over time is negligible.
4. The system has three states: normal, partial failure and total failure.
5. A unit in the normal mode must pass through the partial failure mode.
6. A unit which is replaced or repaired in total failure go directly to the normal mode without passing through the partial failure mode
7. Unit failure and repair rates are constants.
8. Failure rates and repair rates follow an exponential distribution.
9. A repaired unit works as good as new.
10. The system is down when both units are non-operative.

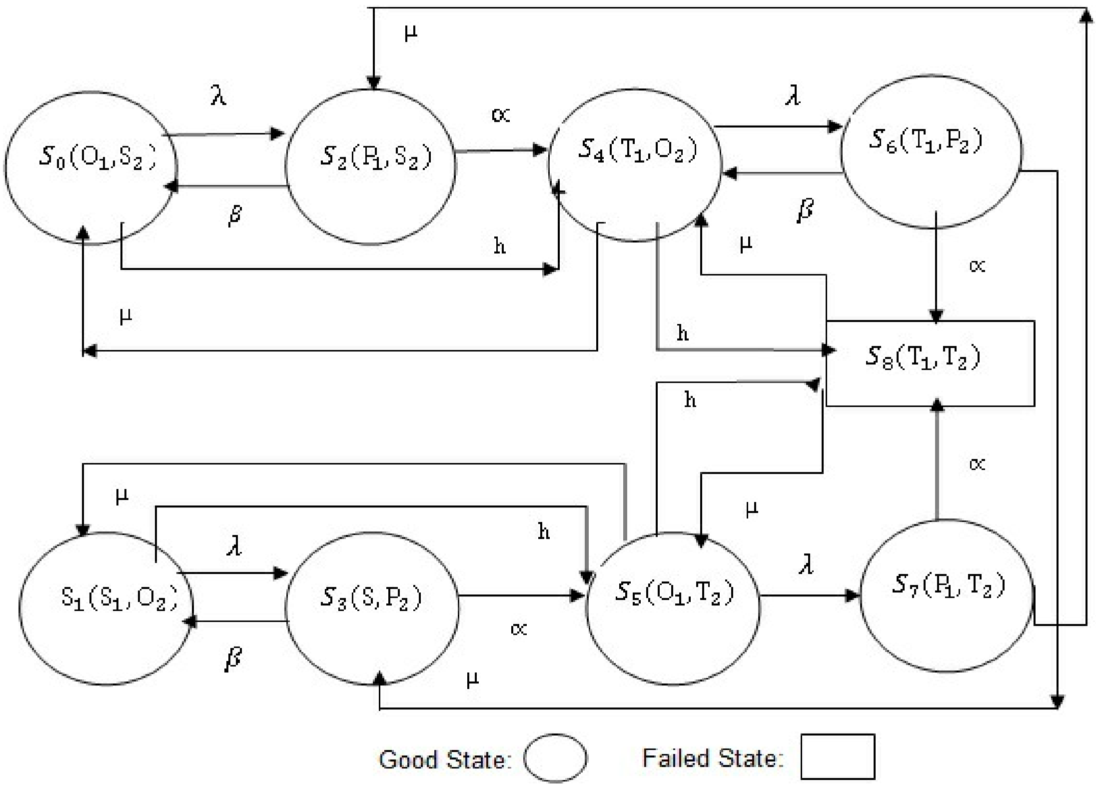

Figure (1) State of the system

Notations

λ : the failure rate of the i th unit from normal mode to partial mode, i= 1, 2.
α : the failure rate of the i th unit from partial mode to total failure mode, i= 1, 2.
β : the repair rate of the i th unit from partial mode to normal mode, i= 1, 2.
μ : the repair rate of the i th unit from total failure mode to normal mode, i= 1, 2.
O_i : the i th unit is in normal mode, i= 1, 2.
S_i : the i th unit is in standby mode . i= 1, 2.
P_i : the i th unit is in partial failure mode, i= 1, 2.
T_i : the i th unit is in total failure mode , i = 1, 2.
$P_i(t)$: Probability that the system is in state S_i at time t, , $(t \geq 0)$, i = [0- 8].

3- Reliability Assessment

The mean time to system failure (MTSF) for the proposed system evaluated using the above-mentioned set of assumptions and method of linear first order differential equations. Let P(t) denote the probability row vector at time t,, the initial conditions for this problem are:

$$P(0) = [P_0(0) \;\; P_1(0) \;\; P_2(0) \;\; P_3(0) \;\; P_4(0) \;\; P_5(0) \;\; P_6(0) \;\; P_7(0) \;\; P_8(0) = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0] \qquad (1)$$

By employing the method of linear first order differential equations for Fig. 1, the following differential equations (2) can be obtained:

$$P'_0(t) = -(\lambda + h)P_0(t) + \beta P_2(t) + \mu P_4(t)$$

$$P'_1(t) = -(\lambda + h)P_1(t) + \beta P_3(t) + \mu P_5(t)$$

$$P'_2(t) = -(\alpha + \beta)P_2(t) + \lambda P_0(t)$$

$$P'_3(t) = -(\alpha + \beta)P_3(t) + \lambda P_1(t)$$

$$P'_4(t) = -(\lambda + h + \mu)P_4(t) + hP_0(t) + \alpha P_2(t) + \mu P_7(t)$$

$$P'_5(t) = -(\lambda + h + \mu)P_5(t) + hP_1(t) + \alpha P_3(t) + \mu P_8(t)$$

$$P'_6(t) = -(\alpha + \mu)\, P_6(t) + \lambda P_4(t)$$

$$P'_7(t) = -(\alpha + \mu)P_7(t) + \lambda P_5(t)$$

$$P'_8(t) = -2\mu P_8(t) + hP_4(t) + hP_5(t) + \alpha P_6(t) + \alpha P_7(t) \qquad (2)$$

The above system of differential equations can be written in the matrix form as shown in equations 3 and 4:

$$P^{*} = Q \times P \qquad (3)$$

Where,

$$Q = \begin{bmatrix} -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 \\ \lambda & 0 & -(\alpha+\beta) & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & \lambda & 0 & -(\alpha+\beta) & 0 & 0 & \mu & 0 & 0 \\ h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & 0 & \mu \\ 0 & h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & \mu \\ 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 \\ 0 & 0 & 0 & 0 & h & h & \alpha & \alpha & -2\mu \end{bmatrix}$$

(4)

Mean Time to System Failure (MTSF)

To calculate the MTSF we take the transpose matrix of Q and delete the rows and columns for the absorbing state, the new matrix is called A. the expected time to reach an absorbing state is calculated from equation 5.

$$MTSF = P(0)(-A^{-1})\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (5)$$

Where,

$$A = \begin{bmatrix} -(\lambda+h) & 0 & \lambda & 0 & h & 0 & 0 & 0 \\ 0 & -(\lambda+h) & 0 & \lambda & 0 & h & 0 & 0 \\ \beta & 0 & -(\alpha+\beta) & 0 & \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & -(\alpha+\beta) & 0 & \alpha & 0 & 0 \\ \mu & 0 & 0 & 0 & -(\lambda+h+\mu) & 0 & \lambda & 0 \\ 0 & \mu & 0 & 0 & 0 & -(\lambda+h+\mu) & 0 & \lambda \\ 0 & 0 & 0 & \mu & 0 & 0 & -(\alpha+\mu) & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 & 0 & -(\alpha+\mu) \end{bmatrix}$$

The steady state Mean Time to System Failure (MTSF) is given by equation 6:

$$MTTF = \frac{a1+a2+a3+a4}{a_5} \qquad (6)$$

Where,

$$a_1 = 2\alpha(h+\lambda)(\alpha+\lambda)$$

$$a_2 = \mu(\alpha+\lambda)(\alpha+\lambda+\mu)$$

$$a_3 = \beta(\alpha+\mu)(\lambda+\mu)$$

$$a_4 = 2\alpha h(\beta+\mu)$$

$$a_5 = (h\alpha + h\beta + \alpha\lambda)(h\alpha + h\mu + \alpha\lambda)$$

When human error is not considered

The steady state Mean Time to System Failure (MTSF) is given by equation 7:

$$MTTF = \frac{a1+a2+a3}{a4} \qquad (7)$$

Where: $a_1 = 2\alpha\lambda(\alpha+\lambda)$, $a_2 = \mu(\alpha+\lambda)(\alpha+\lambda+\mu)$

$$a_3 = \beta(\alpha+\mu)(\lambda+\mu), \quad a_4 = \alpha^2\lambda^2$$

4- Availability analysis

The initial conditions for this problem are the same as for the reliability case:

P(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0] ,the differential equations form can be expressed as shown in equation 8:

$$\begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \\ P_6' \\ P_7' \\ P_8' \end{bmatrix} = \begin{bmatrix} -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 \\ \lambda & 0 & -(\alpha+\beta) & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & \lambda & 0 & -(\alpha+\beta) & 0 & 0 & \mu & 0 & 0 \\ h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & 0 & \mu \\ 0 & h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & \mu \\ 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 \\ 0 & 0 & 0 & 0 & h & h & \alpha & \alpha & -2\mu \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} \qquad (8)$$

In the steady state, the derivatives of the state probabilities become zero, as presented in equation 9.

$$QP(\infty) = 0 \qquad (9)$$

Then the steady state probabilities can be calculated as shown in equation 10.

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty) \qquad (10)$$

Then the matrix form became as displayed in equation 11.

$$\begin{bmatrix} -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 \\ \lambda & 0 & -(\alpha+\beta) & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & \lambda & 0 & -(\alpha+\beta) & 0 & 0 & \mu & 0 & 0 \\ h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & 0 & \mu \\ 0 & h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & \mu \\ 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 \\ 0 & 0 & 0 & 0 & h & h & \alpha & \alpha & -2\mu \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (11)$$

To obtain $P_0(\infty)+P_1(\infty)+P_2(\infty)+P_7(\infty)$, equation 9 must be solved by using the normalizing condition indicated in equation 12.

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+\dots\dots\dots+P_8(\infty)=1 \qquad (12)$$

Substituting equation 12 in any one of the redundant rows in equation 9 yields equation 13.

$$\begin{bmatrix} -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+h) & 0 & \beta & 0 & \mu & 0 & 0 & 0 \\ \lambda & 0 & -(\alpha+\beta) & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & \lambda & 0 & -(\alpha+\beta) & 0 & 0 & \mu & 0 & 0 \\ h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & 0 & \mu \\ 0 & h & 0 & \alpha & 0 & -(\lambda+h+\mu) & 0 & 0 & \mu \\ 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & -(\alpha+\mu) & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (13)$$

The steady state availability A is given by equation 14:

$$A(\infty)=1-p_8=1-\frac{N}{D} \qquad (14)$$

Where: $N=(h\alpha+h\beta+\alpha\lambda)(h\alpha+h\mu+\alpha\lambda)$

$$D=\alpha^2(h+\lambda)^2+\beta\alpha h(h+\lambda)+2\mu^2\lambda(h+\beta)+2\mu\lambda(\alpha+\mu)(\alpha+\lambda+\mu)+2\mu(h+\mu)(\alpha+\mu)(\alpha+\beta)+\mu\lambda h(3\alpha+2\beta)+\mu h^2(\alpha+\beta)$$

When human error is not considered

The steady state availability A (∞) is given by equation 15:

$$A(\infty)=1-p_8=1-\frac{\alpha^2\lambda^2}{D_1} \qquad (15)$$

Where: $D_1=\alpha^2(\lambda^2+2\mu^2)+2\mu\lambda\alpha(\alpha+\lambda+2\mu)+2\mu^2(\beta\mu+\lambda^2)+\alpha(2\mu^3+2\beta\mu^2)+\lambda(2\mu^3+2\beta\mu^2)$

The steady state Busy period is given by equation 16.

$$B(\infty)=(1-(p_0+p_1))=1-\frac{N_1}{D} \qquad (16)$$

Where: $N_1=2\mu^2(\alpha(\alpha+\beta+\mu)+\beta(\lambda+\mu))$

When human error is not considered

The steady state busy period $B(\infty)$ is given by equation 17:

$$B(\infty)=(1-(p_0+p_1))=1-\frac{N_1}{D_1} \qquad (17)$$

Cost analysis: The expected total profit per unit time incurred to the system in the steady-state is given by equation 18:

Profit = total revenue - total cost
PF=RA (∞) - CB(∞) (18)

Where: PF: is the profit incurred to the system,
R: is the revenue per unit up-time of the system,
C: is the cost per unit time when the system is under repair
From equations14, 16, the expected total profit per unit time incurred to the system in the steady-state is given by equation 19:

$$PF=R\times A(\infty)-C\times B(\infty)=R\left(1-\frac{(h\alpha+h\beta+\alpha\lambda)(h\alpha+h\mu+\alpha\lambda)}{D}\right)-C\left(1-\frac{2\mu^2(\alpha(\alpha+\beta+\mu)+\beta(\lambda+\mu))}{D}\right) \qquad (19)$$

From equations 15,17, the expected total profit per unit time incurred to the system in the steady-state is given by equation 20:

$$PF = R \times A(\infty) - C \times B(\infty) = R\left(1 - \frac{\alpha^2\lambda^2}{D_1}\right) - C\left(1 - \frac{2\mu^2(\alpha(\alpha+\beta+\mu)+\beta(\lambda+\mu))}{D_1}\right) \qquad (20)$$

5- Numerical computation

Putting λ=.03, α=.04, *h*=.01, β=.05, μ=.06 in equations 6, 7; 14, 15; 19, 20 we get the following:

1. Table (1): Shows relation between failure rate (λ) and the MTSF of the system (with and without Human failure).
2. Table (2): Shows relation between failure rate (λ) and the steady state availability of the system (with and without Human failure).
3. Table (3): Shows relation between failure rate (λ) and the expected total profit of the system (with and without Human failure).
4. Fig. 2: Shows relation between the failure rate (λ) and the MTSF.
5. Fig. 3: Shows relation between the failure rate (λ) and the steady state availability.
6. Fig. 4: Shows relation between the failure rate (λ) and the expected total profit.

MTSF of the system without Human failure	MTSF of the system with Human failure	λ
4400	476.36	0.01
1483.3	372.53	0.02
856.79	313.97	0.03
608.33	276.84	0.04
480	251.38	0.05
403.09	232.92	0.06
352.38	218.97	0.07
316.67	208.06	0.08
290.26	199.33	0.09
270	192.18	0.1

Table (1): Relation between failure rate (λ) and the MTSF (with and without Human failure)

steady state availability of the system without Human failure	steady state availability of the system with Human failure	λ
0.99759	0.97561	0.01
0.99256	0.96852	0.02
0.98677	0.96239	0.03
0.98101	0.95711	0.04
0.97561	0.95256	0.05
0.97066	0.94862	0.06
0.96618	0.94518	0.07
0.96215	0.94216	0.08
0.95851	0.9395	0.09
0.95522	0.93714	0.1

Table (2): Relation between failure rate (λ)) and availability (with and without Human failure)

The profit of the system without Human failure	The profit of the system with Human failure	λ
978.04	942.13	0.01
958.73	923.87	0.02
942.31	909.4	0.03
928.48	897.74	0.04
916.8	888.19	0.05
906.88	880.24	0.06
898.39	873.53	0.07
891.06	867.82	0.08
884.69	862.89	0.09
879.1	862.89	0.1

Table (3): Relation between failure rate (λ) and the profit (with and without Human failure)

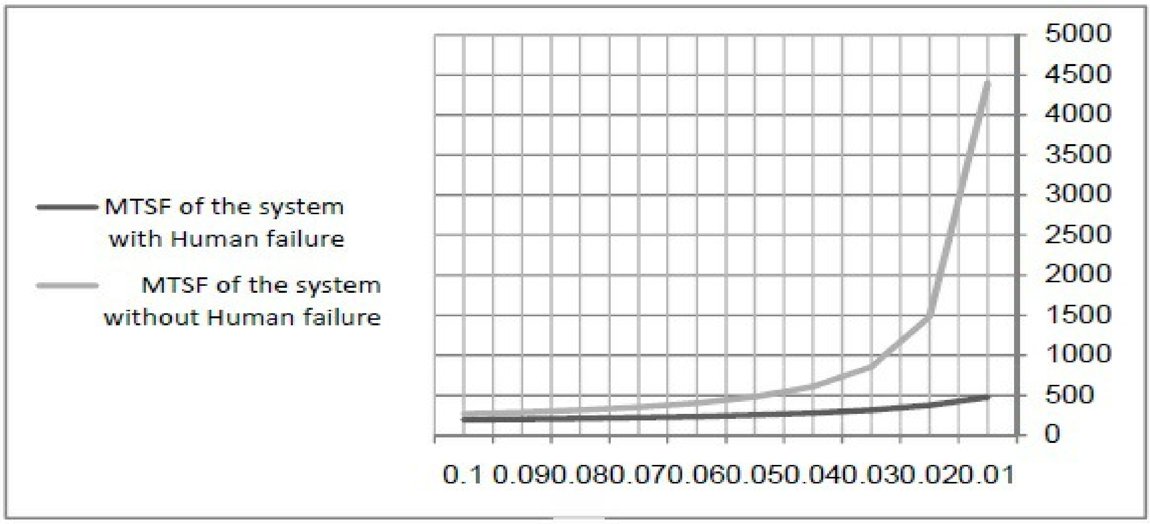

Fig. 2: Relation between the failure rate (λ) and the MTSF

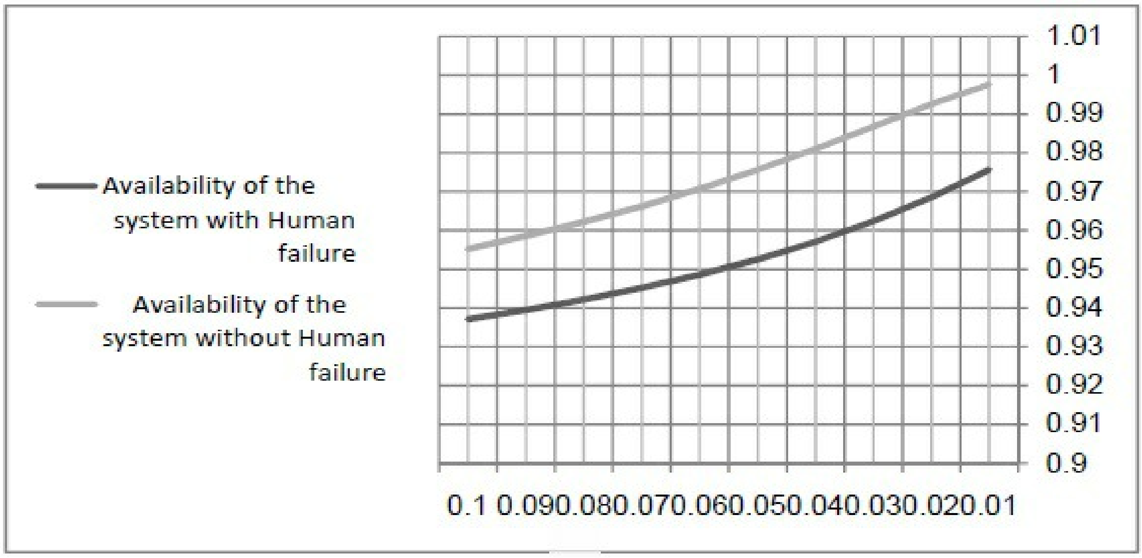

Fig. 3: Relation between the failure rate (λ) and the Availability

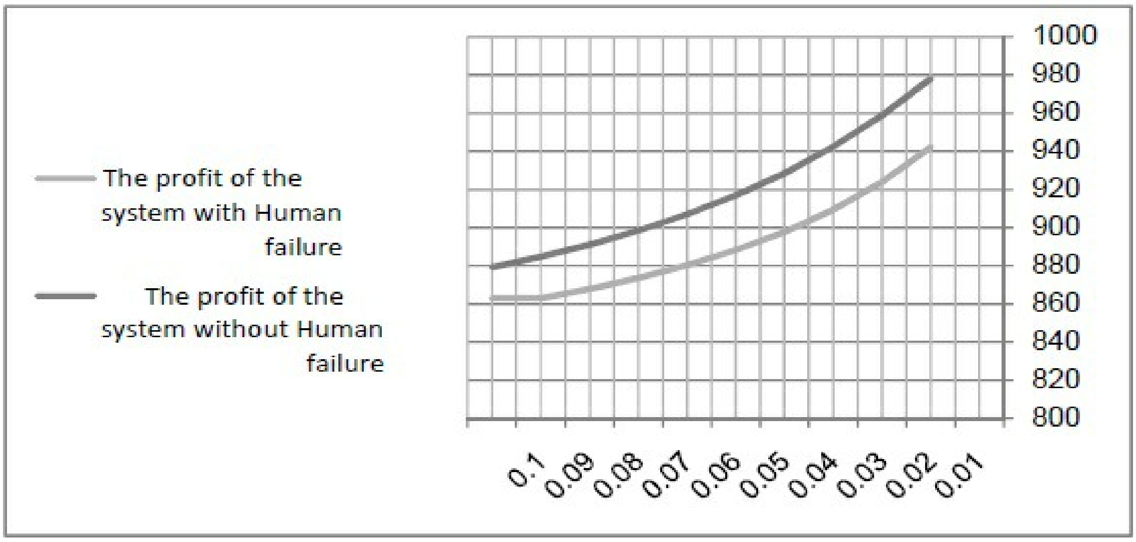

Fig. 4: Relation between the failure rate (λ) and the expected total profit

CONCLUSION

By comparing the characteristic, MTSF, steady state availability and the profit with respect to (λ) for both systems with and without human failure graphically. It was observed that: The increase of failure rate (λ) at constant α=.04,*h*=.01,β=.05,μ=.06, R =1000, C= 100, the MTSF, Availability and the profit function of the system decrease for both systems with and without human failure. It was also observed that the system characteristics without human error are more greater than the system without human failure.

is concluded that the system without human failure is more better than the system with human failure with respect to the MTSF, steady state availability and the profit function.

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