

## Non-normal Distribution of Temperatures in the United States of America during 1895-2016: A Challenge to the Central Limit Theorem

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### ABSTRACT

Statistical arguments show the mean annual temperatures of the United States do not follow the precepts of the Central Limit Theorem (CLT). Calculations of descriptive statistics provided results with a skewness value of .6548 and of kurtosis value of .494 showing a substantial variation of the data suggesting that the temperatures do not follow a normal distribution. We constructed normal probability graphs utilizing the Anderson-Darling goodness of fit test to confirm the normality of the data obtaining an A-D value of 1.716 and  $P < .005$  indicating the mean annual temperatures are not symmetrical. Calculations of density probabilities proved that the configuration of the density probability graph is skewed to the right due to the variation of the temperature values, thus, asymmetrical. Calculations of the objective and subjective evaluations of the temperature values by using a quadratic model yielded an objectivist diagnostics of  $R^2 = 31.7\%$ ,  $PRESS = 81.59$ ,  $VIF$  (Variance inflation factor) = 16.5 and the Durbin-Watson = 1.537. Besides, the subjective residual graphs revealed the temperature data is far from normal. Our results indicate the United States mean annual temperature values are challenging the suppositions of the CLT.

**Keywords:** Central Limit Theorem, normal distribution, temperature variations, Durbin-Watson (D-W) statistics, Anderson-Darling goodness of fit test, United States ambient temperatures.

### 1. Introduction

The goal of this research is to assert that global warming is causing much variation of the ambient temperatures of the United States of America. According to the U.S. Global Change Research Program there is an increase in the earth's atmospheric and oceanic temperatures widely predicted to occur due to an increase in the greenhouse effect resulting especially from pollution. In view of that, the year of 2016 is officially the new hottest year on record, edging out previous record holder 2015 by 0.07°F. It is the third year in a row that global average surface temperature set a new record, and the fifth time the record has been broken since the start of the twenty-first century [1-4]. As warrant by the majority of scientists, global warming is a fact. On the consensus of scientists who affirm global warming is anthropogenic (man-made). Cook *et al* (2016) [5-6] affirm that "depending on exactly how you measure the expert consensus, it's somewhere between 90% and 100% of scientists that agree, humans are responsible for climate change [7] with most of the studies finding 97% consensus among publishing climate scientists." Aside from the previous statements, climatic changes caused by global warming is causing much variation and a gradual increase on the overall temperature of the earth's atmosphere generally attributed to the greenhouse effect caused by increased levels of carbon dioxide, chlorofluorocarbons, and other pollutants [8]. For this reason, global warming is causing much variability in the world ambient temperatures. This phenomenon is challenging the Central Limit Theorem (CLT), which affirms that the statistical distribution of the mean of the means tends to the normal distribution [9]. In fact, the CLT is one of the most notable results of the theory of probability. The CLT sustains that the distribution of the sample means approaches the normal distribution invariant of the population under study, that is, regardless of the form of the original parental population. The

CLT was first postulated by De Moivre, who initiated it in 1733, and by Laplace who formalized in 1820. In simple terms, the CLT explains that the statistical distribution of the sample means tends to the normal distribution, because if we draw random samples of the same size from the same population and construct a histogram, it will form a bell-shaped distribution. According to the CLT this supposition is correct regardless of the form of the original parent distribution [10-11]. Additional postures about the CLT says the sampling distribution of the mean of any independent, random variable will be normal or nearly normal, if the sample size is large enough [12].

Here, however, there is a dilemma when we ask ourselves "how large is enough." Agreeing with this information, the answer depends on the requirements for accuracy, because the more closely the sampling distribution needs to resemble a normal distribution, the more sample points will be required [13]. However, from our standpoint, this assertion is right, when we talk about infinite or unrestricted samples sizes, but it cannot apply to finite world ambient temperatures, because temperature records have only been disclosed since the eighteen hundreds. So, from this perspective, this requirement of the Central Limit Theorem (CLT) cannot be satisfied, because the sample world mean annual temperatures is restricted. Similarly, according to the above source of information, another factor related to the sample size is related to the shape of the underlying population, because the more closely the original population resembles a normal distribution, the fewer sample points will be required. In view of that, in practice, some statisticians say that a sample size of 30 is large enough, when the population distribution is roughly bell-shaped. Besides, other statisticians and investigators recommend a sample size of at least 40, but if the original population is distinctly not normal (e.g., is badly skewed, has multiple peaks, and/or has outliers), researchers

recommend the sample size to be even larger. Again, here there is ambiguity, when we talk about global ambient temperatures, because the sample size of world temperatures is limited. Consequently, these affirmations challenge the suppositions of the CLT regarding world ambient temperatures. In this way, according to a sound intellectualism, on the suppositions of the CLT, there are uncertainties, when we talk about global temperature values.

$$Z = \frac{(X_1 + X_2 + \dots + X_n) - n\mu}{\sigma\sqrt{n}} = \frac{[(\sum_{i=1}^n X_i) - n\mu]}{\sigma\sqrt{n}} \quad (1)$$

This function is approximated to the typified normal variable  $N(0,1)$  by optimizing the quality of the approximation as  $n$  increases. This result proves the statistics of the sample mean given below:

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \sum_{i=1}^n X_i / n \quad (2)$$

Which is approximately distributed as a variable:

$$N(\mu, \sigma/\sqrt{n}) \quad (3)$$

Otherwise, equivalent to:

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \quad (4)$$

Which is approximately distributed as a variable  $N(0, 1)$

In more simple terms, an euphonic explanation of the CLT is given as follows: Suppose that  $X_1, X_2, X_3, \dots, X_n$  are random

This is because the population of world ambient temperatures is not normally distributed, as contended by this research.

## 2. Mathematical demonstrations of the Central Limit Theorem

The mathematical demonstration of the Central Limit Theorem can be given as follows. Thus, if  $X_1, X_2, \dots, X_n$  are independent random variables, with identical probability, with mean value  $\mu$  and variance  $\sigma^2$ , then, the distribution of the random variable  $Z$  is described as follows [14]:

independent variables and distributed identically with mean  $\mu$  and finite variance  $\sigma^2$  then, the random variable  $Z_n$  is defined as follows [15]:

$$Z_n = \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \quad (5)$$

Therefore, the form of the  $Z_n$  distribution converges to the shape of the normal standard distribution as,  $n \rightarrow \infty$ , keeping  $\sigma$  Constant.

However, if we apply this concept to a random sample of  $n \geq 32$ , nonetheless, there exists the possibility of the presence of experimental error (the difference between a measurement and the true value or between two measured values), when we deal with global ambient temperatures. This is because of the great variation that exists with the global ambient temperatures. However, aside from that, using the logic of the CLT, combined with the application of experimental design, in collecting samples from that same population and by calculating the mean of the means, the distribution of the simple means will tend to follow the normal distribution with standard deviation,  $s = 0$  or very close to zero, thus controlling the experimental error. According to the previous discussion, the CLT explains the ubiquity of the normal distribution in a bell-shaped form within the domain of the measurements [16-17].

Nonetheless, agreeing with the results of this study, when we try to apply the Central Limit Theorem (CLT) to the population of ambient temperatures, at world level or to the mean annual temperatures of the United States, there seems to be uncertainties, because of the great variability of the data due to the effects of global warming. (Here, as a curious observation, De Moivre who initiated the CLT and Laplace who formalize it [18] never imagined that anthropogenic global warming and concomitant climatic changes caused by the aftermath of the industrial revolution, was going to challenge their Central Limit Theorem thesis).

### 2.1. Global warming and temperature variations caused by anthropogenic vehicle and industrial activities are questioning the Central Limit Theorem principles.

We contend the Central Limit Theorem has applications to resolve theoretical problems, but not practical problems, when dealing with the effects of global warming and the concomitant climatic distortion that is causing ambient temperature variations [19]. This is because climatic changes are causing an increase in the world temperatures and much climatic variations, expressed as, too much heat and cold, (but more heat than cold), especially in New England, due to the deformation of the Jet Stream, a fast flowing, narrow, meandering air currents located near the altitude of the tropopause and with westerly winds) [20]. Then, due to these unfortunate circumstances, technically speaking, there is no solution to this problem. The only way to solve this difficult would be by increasing the sample size. Ominously, in relation to global ambient temperatures, the size of the random sample is limiting, because there cannot be a sample size that would be boundless, in order to satisfy the CLT dictates. This assertion is sustained by the fact the global ambient temperature records began to be documented until about the eighteen hundreds [21].

The problem that we are witnessing up to this point in time, about anthropogenic climatic changes is the result of tying the world economy to the indiscriminate use of petroleum and its derivatives. This is because, in those incorrect attitudes, there is much economic, political, cultural and social power; situations that do not concur with the use of clean energy technologies that do not proceed with that brashness.

Similarly, Zoheir *et al* discuss the macroeconomic impacts of oil price volatility and its mitigation and resilience [22]. The anthropogenic activities are causing greenhouse gases that are trapping the sun's heat in the planet's lower atmosphere due to the greater transparency of the atmosphere to visible radiation from the sun than to infrared radiation emitted from the planet's surface [23-25]. In order to mitigate the experimental error caused by temperature variations due to global warming, which is interfering with the probability calculations of the response variable, it is necessary to increase the sample size (because nothing can be done to stop the global warming). Unfortunately, in practice, the size of the ambient temperatures can never be infinite, because, evidently, there cannot be sample sizes of ambient temperatures that can be limitless or infinite. This reasoning is sustained by the NASA GISS organization, which affirms that global temperature records exist only since 1850. Moreover,

in the United States of America, temperature records exist just from 1895 [26]. Consequently, the price that would have to be paid in increasing the sample size, without doing anything to stop the global warming, would be a strong impact on the world economy and concomitant inflationary pressures. Undoubtedly, this will happen, because that would be the same as attacking the effect (anthropogenic global warming), but not the real cause of the problem (the indiscriminating use of fossil fuels).

### 3. Methodology

The methodology used in this study consisted in processing the United States mean annual temperature values of a sample data of 122 years, for the period 1895 to 2016. These values were apportioned by the National Oceanic Atmospheric Administration (NOAA) [27], as shown in Table 1 below. This table depicts the time, in years, and the mean annual temperatures expressed in degrees Fahrenheit (°F).

**Table 1:** Table depicting the time, in years, beginning in 1895 to 2016 and the mean annual temperatures, expressed in degrees Fahrenheit of the United States of de America.

Time (Years)	Mean annual temperature (°F)	Time (Cont.)	Mean annual temp. (Cont.)	Time (Cont.)	Mean annual temp. (Cont.)
1895	50.34	1936	52.15	1977	51.05
1896	51.99	1937	51.55	1978	50.88
1897	51.56	1938	53.18	1979	52.39
1898	51.43	1939	53.26	1980	53.12
1899	51.01	1940	51.89	1981	51.35
1900	52.77	1941	52.66	1982	51.88
1901	51.87	1942	51.84	1983	51.98
1902	51.59	1943	52.07	1984	51.3
1903	50.62	1944	51.83	1985	53.32
1904	51.16	1945	51.75	1986	53.33
1905	51	1946	52.95	1987	52.63
1906	51.73	1947	51.92	1988	51.84
1907	51.48	1948	51.61	1989	53.51
1908	52.08	1949	52.02	1990	53.16
1909	51.43	1950	51.39	1991	52.6
1910	52.42	1951	51.12	1992	51.26
1911	52.03	1952	52.27	1993	52.87
1912	50.23	1953	53.37	1994	52.65
1913	51.54	1954	53.33	1995	51.89
1914	51.84	1955	51.69	1996	52.2
1915	51.45	1956	52.34	1997	54.23
1916	50.85	1957	52.04	1998	53.88
1917	50.06	1958	51.93	1999	53.27
1918	51.87	1959	52.11	2000	53.7
1919	51.55	1960	51.44	2001	53.21
1920	51.07	1961	51.87	2002	53.26
1921	53.8	1962	51.9	2003	53.1
1922	52.03	1963	52.26	2004	53.64
1923	51.64	1964	51.67	2005	54.25
1924	50.59	1965	51.69	2006	53.65
1925	52.52	1966	51.49	2007	52.29
1926	51.95	1967	51.76	2008	52.39
1927	52.15	1968	51.32	2009	52.98
1928	51.92	1969	51.5	2010	53.18
1929	50.85	1970	51.61	2011	55.28
1930	51.98	1971	51.66	2012	52.43
1931	53.54	1972	51.37	2013	52.54
1932	51.73	1973	52.29	2014	54.4
1933	52.99	1974	52.26	2015	54.92
1934	54.1	1975	51.5	2016	54.92
1935	51.9	1976	51.47		

Source: NOAA National Centers for Environmental information, Climate at a Glance: U.S. Time Series, Maximum Temperature, published May 2017, retrieved on June 4, 2017 (from ref. 13).

The procedure consisted in reviewing the descriptive statistics to summarize the given data set of temperatures to form a representation of the sample of the mean annual temperatures aimed to check for the symmetry of the data. This information is depicted in figure 1 (please see the results section).

Subsequently, the method included a structuration of a normal probability graph basing the criterion on the Anderson-Darling goodness of fit test, and the P-value, to attest if the temperature values were following the normal distribution [28-29]. This diagnostic was based on the fact that, as the A-D value approaches zero, it means the data follows the normal distribution, and vice versa, too. This situation is shown in figure 2 (please see the results section).

Additionally, the method performed a test of hypothesis using the resulting P-value shown in figure 2, by stating the null hypothesis as  $H_0$ : affirming that the data followed the normal distribution, against the alternative hypothesis,  $H_A$ : the data did not follow the normal distribution. Then the procedure compared the P-value against the alpha value of  $\alpha = 0.05$ , to see if the null hypothesis was rejected (A small P-value of typically  $\leq 0.05$  indicates a strong evidence against the null hypothesis, therefore rejecting it). This method was used to conclude that, even though, the research used a sample size of  $n = 120$  mean annual temperatures, the data still did not follow the normal distribution. This process was done to prove the temperature data was not in agreement with the suppositions of the CLT. Moreover, the methodology calculated the density probabilities and plotted these values to observe the configuration of the data, that is, to revise for the skewness and symmetry of the temperature data. This information is depicted in figure 3 (please see the results section).

Furthermore, the methodology included an objectivistic assessment (direct perception of statistical information) and

$$D = \frac{\sum_{i=2}^n (e_i - e_{i-1})^2}{\sum_{i=2}^n e_i^2} \quad (6)$$

The D stands for the D-W statistics,  $e_1, e_2, \dots, e_n$  are sample residuals ordained according to time and  $n$  is the number of observations.

In this respect, when using equation (6) the ideal value of the D-W should be 2, but if its value is greater than 2, it means positive autocorrelation faults. Also, if the D-W is less than 2, it means negative autocorrelation. (Serial autocorrelation is a situation that upsets the objectivistic evaluation of the candidate model). This assertion is manifested by doing a hypothesis testing. For example, the decision rule is that, if  $D < d_L$ , that is,  $1.5368 < 1.654$ , then, the errors are positively autocorrelated against  $D > d_U$ , that is,  $1.5368 > 1.694$ , the errors are not positively autocorrelated. To make this test, the procedure consulted the Durbin-Watson table with  $n = 100$  (because there are no values for 119) and  $k = 1$ , which yielded  $d_L = 1.654$  and  $d_U = 1.694$ . So, the decision rule was that the errors were positively autocorrelated.

About the subjectivist graphical residual evaluation of the chosen regression model of the ambient temperatures, this is a complementary or indirect method of diagnosing the utility of the selected model. In this way, a residual plot is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. The Minitab

subjectivist evaluation (indirect perception of graphical information) of the temperature data. For example, the objectivistic assessment included the calculation of the regression equation, the coefficient of determination ( $R^2$ ) [30], the standard error of estimate ( $s$ ) [31], the predicted sum of squares ( $PRESS$ ) [32], the variance inflation factors (VIF) [33], the Durbin-Watson statistics [34] and the analysis of variance (ANOVA) [35]. In this instance,  $R^2$  is used to measure how good the regression model fits the data; it estimates the population coefficient  $\rho$ . In this context, as the  $R^2$  value approaches 100%, the data fits the model. Similarly, the value of  $s$  gives an indication of the amount of experimental error that can exist. Small values of  $s$  are desirable. About the statistics  $PRESS$ , this is a function used to evaluate the utility of the selected model. Accordingly, large values of  $PRESS$  produce poor candidate models with much variation and vice versa. The function variance inflation factors (VIFs) quantifies how much the variance is inflated, as compared to when the predictor variables are not linearly related. In this respect, an acceptable value of VIF is about 5. If this value is exceeded, there would be multicollinearity problems. Also, about the logic of Minitab computer program, a  $VIF = 1$ , means the VIF is not correlated, but if  $1 < VIF < 5$ , the VIFs are moderately correlated. Dissimilar, if the VIFs  $> 5$  to 10, the data is highly correlated [36-37]. From this perspective, collinearity causes all kinds of problems, especially in the calculations of the coefficient estimates of the multiple regression equation. Moreover, standard errors of the estimate tend to be large, situation which lead to large confidence intervals. Besides, multicollinearity will lead to the acceptance of null hypotheses, because type II error will be large.

About the Durbin-Watson (D-W) statistics, this function is used to check, if there were time series autocorrelation problems [38]. The D-W statistics is defined below:

computer program gives four types of graphs to assess the utility of the regression model as shown in figure 5 (please see results section).

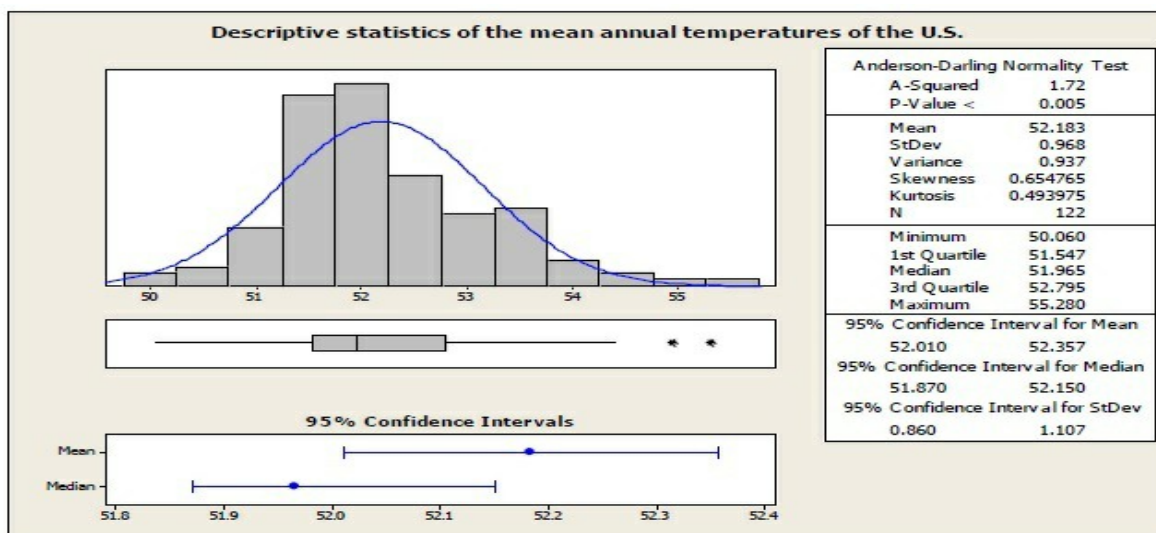
For example, in the normal probability graph, for a model to be good, all the points must be close to the least square regression line [39]. If this is not the case, the chosen regression model is not good, with much variation. Similarly, on the fitted values graph, there must be randomness and there must be the same number of positive and negative residuals, otherwise, there will be serial autocorrelation problems. In this respect, to be more precise serial correlation occurs when the observations of the error terms  $\varepsilon_i$  are correlated. This drawback results in a countless of problems. For instance, autocorrelation will cause inefficient ordinary least squares estimates, it will give exaggerated goodness of fit statistically significant and will give false values of  $t$ . Likewise, the graph of the residual versus orders, it is related to the order in which the data was collected and is used to find nonrandom errors, especially of effects related to time. Finally, the histogram graph of residual must contest a bell-shape curve, for the model to be good. The interpretation of the ANOVA data of figure 4 (please see the results section) shows a  $p$ -value  $\leq \alpha$  (e.g.,  $\alpha = 0.05$  or  $0.01$ ) which means that

the differences between some of the means are statistically significant and the null hypothesis is rejected and the conclusion is the population means are equal. However, if  $p\text{-value} > \alpha$ , it means the differences between the means are not statistically significant and there is not enough evidence

to reject the null hypothesis that the population means are all equal.

#### 4. Result/Findings

The descriptive statistics of the mean annual temperatures of the United States are shown in figure 1.

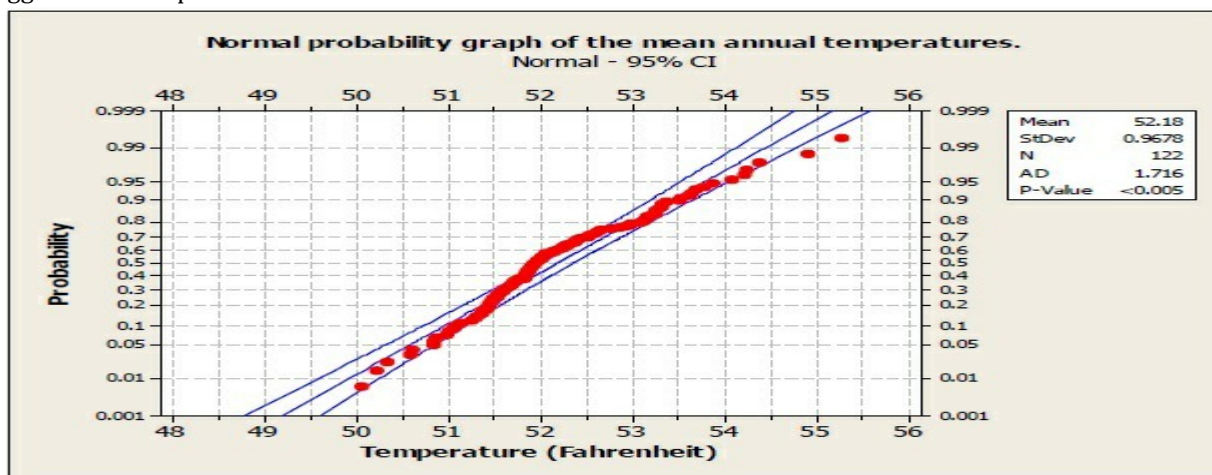


**Figure 1:** Schematic representation of the descriptive statistics of the mean annual temperatures of the U. S.

This figure 1 shows the values of the mean annual temperature of 52.183 oF, a standard deviation of 0.968, variance equal to 0.937, a skewness equal to .6548, kurtosis equal to .49379, median, equal to 51.965 and so on. The values of the skewness of .6548 and the value of the kurtosis of .49379 and the discrepancy of the mean of 52.183 and the median value of 51.965 flag much variation of the mean annual temperatures. Also there are two outliers and wide confidence intervals of the mean, median and standard deviation, situations which contributed much variability to the temperature data.

About the results of the normal probability graph (Figure 2), the Anderson-Darling goodness of fit test equal to 1.716 and the P-value < .005, precluded the temperature values were following the normal distribution. The interpretation of these results is based on the fact that, as the A-D value approaches zero, it means the data follows the normal distribution. However, in this instance, the A-D value of 1.716 is far away from zero, thus flagging experimental noise. Also, the P-value < .005 is much less than the chosen value of  $\alpha = 0.05$ . These results suggest the temperature data is not normal. To

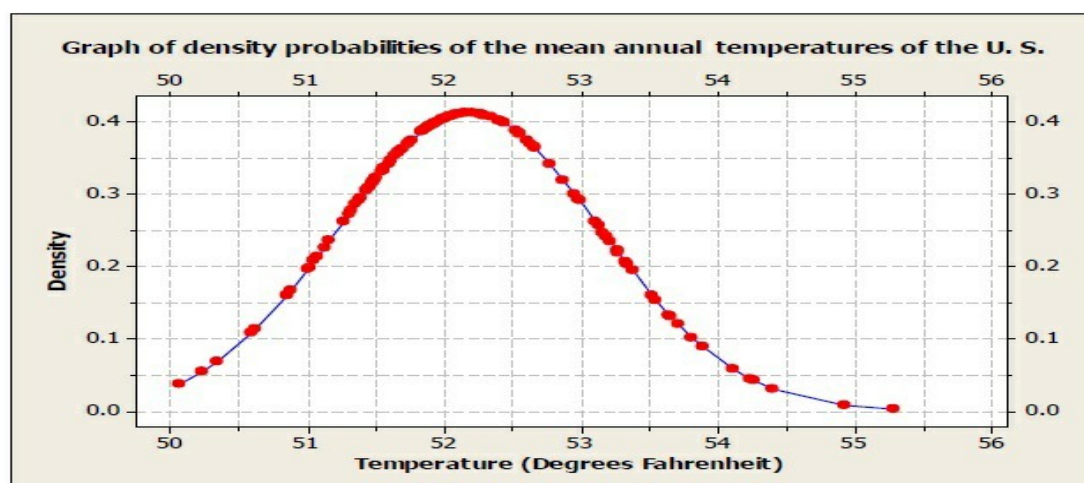
withstand these results, the procedure made a test of hypothesis using the resulting P-value < .005, by stating the test of hypothesis as follows: the null hypothesis,  $H_0$ : follows the temperature data, against the alternative hypothesis,  $H_A$ : the data do not follow the normal distribution. This being so, since the P-value < .005 was much smaller than alpha value of  $\alpha = 0.05$ , thus,  $H_0$ : was rejected. This finding indicates strong evidence against the rejection of the null hypothesis. This logic was used to conclude that, even though, the research used a sample size of  $n = 122$  mean annual temperatures, the data still did not follow the normal distribution, as it should be. According to these results, there is no consensus with the suppositions of the CLT, which contends that as the mean sample size n increases; the distribution of temperatures of the sample means approaches the normal distribution. This contention is exemplified by analyzing equation (1) or (2), because it is seen that, as the sample size n increases the distribution resembles the normal bell-shape. This situation, of course, did not occur here. Figure 2 below shows the normal probability graph with the A-D = 1.716 and P-value < .005.



**Figure 2:** Normal probability graph of the mean annual temperatures of the United States, with an A-D value of 1.716 and a P-Value < 0.005.

Regarding to the calculations of the density probabilities, the corresponding graph is shown in Figure 3 below. The results of this graph clearly show the temperature values are slanted

to the right. This finding witnesses much variation of the mean annual temperatures; circumstances that show lack of symmetry of the mean annual temperature values.



**Figure 2:** Density probability graph of the mean annual temperatures of the United States. As seen in this figure, the distribution of the temperature values is much skewed to the right.

Further, the results of the objective and subjective diagnoses identified a quadratic regression model. These results are depicted in figure 4 below.

The regression equation is:

$$\text{Mean annual temperature} = 51.7 - 0.00560 (\text{Index, Years}) + 0.000164 (\text{Index, Years})^2$$

Predictor	Coef	SE Coef	T	P	VIF
Constant	51.7058	0.2227	232.22	0.000	
Index (years)	-0.005604	0.008357	-0.67	0.504	16.251
Index squared	0.00016368	0.00006582	2.49	0.014	16.251

s = 0.806393 R-Sq = 31.7% R-Sq(adj) = 30.6% PRESS = 81.5906 R-Sq(pred) = 28.01%

Analysis of Variance Table (ANOVA)

Source	DF	SS	MS	F	P
Regression	2	35.959	17.979	27.65	0.000
Residual Error	119	77.382	0.650		
Total	121	113.341			

Unusual Observations

Obs	(years)	media anual	Fit	SE Fit	Residual	St Resid
23	23	50.0600	51.6635	0.1093	-1.6035	-2.01R
27	27	53.8000	51.6738	0.1023	2.1262	2.66R
37	37	53.5400	51.7225	0.0984	1.8175	2.27R
40	40	54.1000	51.7435	0.0997	2.3565	2.94R
118	118	55.2800	53.3235	0.1886	1.9565	2.50R

R denotes an observation with a large standardized residual.

Durbin-Watson statistic = 1.53677

**Figure 4:** Printed schematic representation showing the objectivistic results.

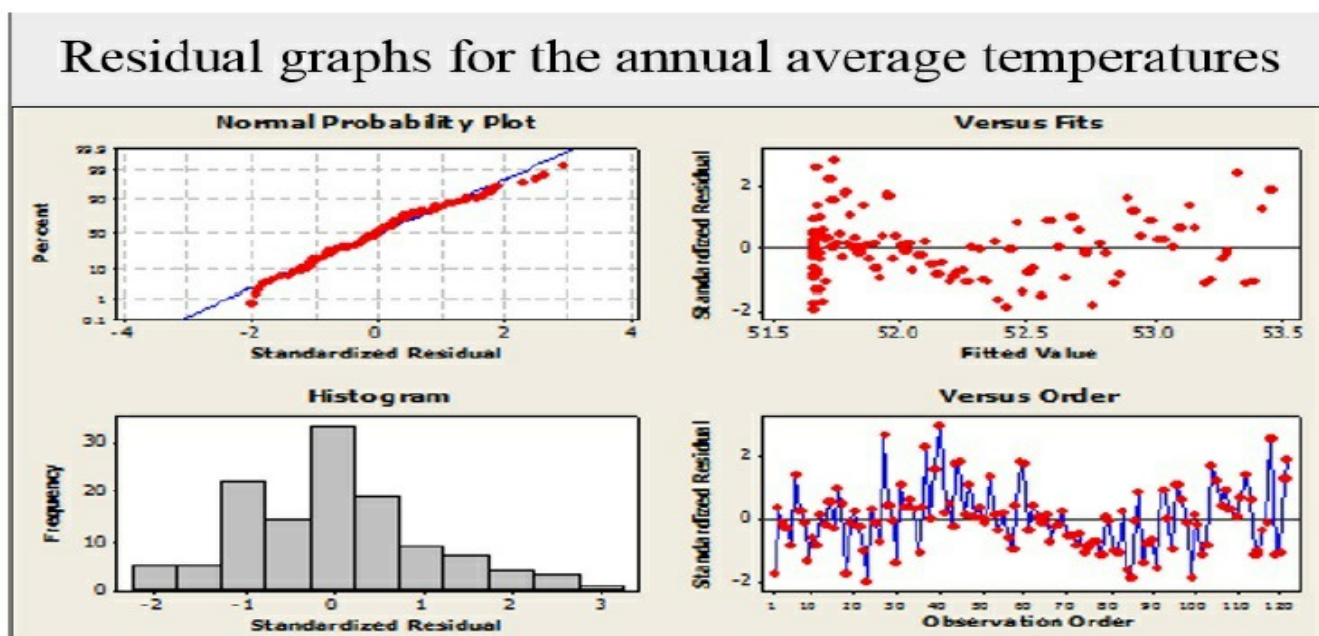
Figure 4 above shows the resulting objective outcomes. For example, it shows the regression equation. It also shows the coefficient of determination  $R^2$  (a statistic that will give some information about the goodness of fit of a model) equal to 31.7% and the standard error of estimate,  $s$  (a measure of the accuracy of predictions made with a regression line) equal to .8064 and the *PRESS* value (*PRESS* or predicted sum of squares evaluates the model's predictive ability; the smaller the *PRESS* value, the better the model's predictive ability) of 81.5906. It

also shows the ANOVA table and the unusual observations. Finally, this figure shows the Durbin-Watson (D-W) statistics (a number that tests for autocorrelation in the residuals from a statistical regression analysis). The resulting value of  $R^2$  is far below the ideal value of 100%, and the *PRESS* value of 81.59 is too big. These outcomes point to much variation and much experimental error. Also the D-W value of 1.537 is distant from the ideal value of 2. All these outcomes do point

to much variation and experimental error (random errors caused by environmental conditions, as global warming).

About the residual graphs of figure 5, the normal probability plot shows many points distant from the least squares line. Likewise, about the fitted values graph there is no randomness

of the data, as it should be; besides, there are not the same numbers of positive and negative residuals, thus flagging positive autocorrelation (an undesirable relationship between two variables in which both variables move in tandem). Also, in the histogram graph, the data is skewed to the right and so on



**Figure 5:** Figure showing the residual plots of normal probability graph, the fitted value graph, the residuals histogram and the observation order graph.

## 5. Discussion

According to the results of this study, there is no agreement on the suppositions of the Central Limit Theorem and the values of the mean annual temperatures of the United States. To support this contention, this research will give a listing of evidences to assert such disagreements. For example, about the results of the descriptive statistics of Figure 1, the skewness and kurtosis values of .6548 and .494 indicate much variation of the mean annual temperatures (the ideal values of the skewness and kurtosis is zero or very close to it, to preclude variation, and thus asymmetry). These occurrences suggest the temperatures do not trail the normal distribution. Moreover, the discrepancy between the values of the mean of 52.183 and the median of 51.965, again, indicate lack of symmetry. Likewise, according to Figure 2, the value of the goodness of fit test of the Anderson-Darling of 1.716 and  $P$ -value  $< .005$  clearly indicate serious autocorrelation problems (that give to experimental errors), which indicate the temperature data does not follow the normal distribution. Evidently, these findings say the mean annual temperatures of the United States are not in agreement with the Central theorem Limit postulates. Besides, the analysis of figure 3 of the distribution of density probability of the mean annual temperatures of the United States shows much skewness to the right. This finding provides much variation of the mean annual temperatures; circumstances that show lack of symmetry of the mean annual temperature values. Additionally, on the results of the objectivistic and subjectivist outcomes (figure 4), which identified a quadratic regression model, showed much variation and much experimental error. For example, the VIF values of 16.251 indicate the existence of collinearity (high correlations between two or more predictor variables; a condition that skews the results in a regression model), which cause autocorrelation problems. As

mentioned previously, from the experimental design point of view, first order autocorrelation indicate the presence of experimental errors, which compromise the desired results. The presence of autocorrelation in applications of time series analyses has many deleterious impacts that affect the regression prediction model and the veracity of the outcomes. In this way, the interpretation of a faulty objectivistic evaluation that involves the coefficient of determination  $R^2$ , the standard error of estimate  $s$ , the *PRESS* and the mean quadratic error, can be misleading. Added to this problem, the Durbin-Watson statistics of 1.537 is distant from the ideal value of 2; situations that flag asymmetry of the data. In addition, figure 5 shows many observations, with large standardized residuals; situations that give to variation and, thus, to experimental faults. All these consequences suggest much variation and inaccurate results. Furthermore, about the subjective residual evaluation of figure 5, it shows additional drawbacks. This contention is supported by analyzing the normal probability graph with many points outside the least square line. Too, the graph of the fitted values lacks randomness. Also, the histogram graph is skewed to the right. Finally, on the issue of the Central Limit Theorem, which generally affirms that, as the size of the sample means increases without bounds, the mean sampling distribution will approach the normal distribution, this assertion is right. However, when we talk about the U. S. ambient temperatures, this supposition does not apply, because there is much temperature variations and because the mean sample size of temperatures is finite (up to this point in time, a maximum sample of only 167 years for world ambient temperatures and only 122 for the U. S.). From this perspective, this requirement of the CLT cannot be satisfied.

In conclusion, rendering to the contentions of this study, the precepts of the Central Limit Theorem cannot be fulfilled. This

is because the sample size of the world (and of the U. S.) mean annual temperatures is restricted, and because the increase of the earth's average atmospheric temperature is causing much variation of the mean annual temperature values of the United States of America. Indeed, the anthropogenic global warming is not only affecting the climate, the economy, the health, the socio-political systems and so on, but it is also creating a myriad of adverse effects and challenging the statistical rules of the Central Limit Theorem.

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