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The Spacetime Of A Cosmic String With Inner Structure In Gauss-Bonnet Gravity

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ABSTRACT

In this paper we obtain the spacetime of a cosmic string with inner structure of r_0 radius, using modified gravity of Gauss-Bonnet *(GB)*. The Gauss-Bonnet gravity modifies the Einstein-Hilbert scalar incorporating higher-order terms on action. We analyzed the string's space-time inside and outside of her core, inside $r < r_0$, we will assume that the energy distribution of the cosmic string is constant, $\sigma = \sigma_0$, and outside null, $\sigma = 0$, for both we determined the solution by using a specific form of the Gauss-Bonnet scalar, $f(G) = \propto G^n$, α being a constant and *n* an integer.

Keywords: space time, cosmic string, gauss-bonnet gravity.

1. Introduction

Einstein's theory of General Relativity give us a coherent description of space, time and gravity at the macroscopic level. It is formulated in such a way that space and time are dynamical quantities determined together with the distribution and motion of matter and energy. As a consequence, the Einstein's theory of gravitation it made possible to build the first mathematical model of the universe, the standard Big Bang model, which matched the available cosmological observations until recently (S. Weinberg, 1972). However, in the last thirty years several shortcomings of Einstein's theory were found and physicians began wondering whether General Relativity is the only fundamental theory capable of successfully explaining the gravitational interaction. This new point of view comes mainly from the study of cosmology. The presence of the Big Bang singularity, together with the flatness, horizon, and monopole problems led to the realization that the standard cosmological model based on Einstein's Gravity and on the Standard Model of particle physics is inadequate to describe the universe at extreme regimes (A.H. Guth, 1981). Other reasons to modify General Relativity are provided by the attempt to fully incorporate Mach's principle into the theory, this one contains only some of Mach's ideas and admits solutions that are explicitly anti-Machian, such as the Godel universe (K. Godel, 1949).

The increasing bulk of data accumulated over the past few years has made the way for a new cosmological model usually called ΛCold Dark Matter or more simply ΛCDM model (G. Magnano *et al*, 1990; Magnano *et al*, 1987). These data indicate that the universe is dominated by an unknown fluid with negative pressure commonly referred to as Dark Energy, which drives the accelerated expansion. An overwhelming number of papers appeared following these observational pieces of evidence, which present a large variety of models attempting to explain the cosmic acceleration, among them, the simplest explanation would be the well known cosmological constant Λ. But the ΛCDM model fails egregiously in explaining why the inferred value of Λ is so tiny (120 orders of magnitude lower) in comparison with the typical value of the vacuum energy density predicted by particle physics (Sahni & Starobinsky, 2000).

The inability to a satisfactory explanation to these problems, and owing too to the lack of a quantum theory of gravity capable to unify interactions and particles, led to a scenario that many others alternative theories to General Relativity were created: *f(R)* Theories of Gravity (Sotiriou & Faraoni,nd; Fujii & Maeda, 2003), Gauss-Bonnet Theory(*GB*) (li *et al*, 2007) and most recently Horava-Lifshitz gravity (P. Horava, 2009), and others (Shin'ichi & Sergei, 2011). In modified *f(R)* gravity, the Einstein-Hilbert action is modified and generalized by the substitution of the Ricci scalar to a generalized function, *f(R)*, *R* is replaced by *f(R)*, *R* being the Ricci scalar. The Horava-lifshitz theory forwards to a direction that renormalized the General Relativity, since the Lorentz invariance doesn't be validity in higher energies. The Gauss-Bonnet Gravity, or *f (G)* gravity, is an enrichment modified gravity model built by contraction of the Riemann tensor. In addition to being motivated by fundamental physics, these theories has been the subject of great interest in cosmology because they naturally exhibit an in inflationary behavior capable of overcoming the shortcomings of the Standard Big Bang model (Duruisseau & Kerner, 1983). In the context of *f(G)* gravity, similarly what happens in *f(R)* gravity, the Ricci scalar is replaced by a function *f(G)*, *G* being a topological invariant in four dimensions built by the contraction of the Riemann and Ricci tensor, there exists a De Sitter point that can be used for cosmic acceleration, in this theory, there are no problems with the Newton laws,

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instabilities and anti-gravity regime (Banijamali & Fazlpour, 2012). Another important aspect of the Gauss-Bonnet gravity is that even in the vacuum spherically symmetric background the Gauss-Bonnet scalar takes a non-vanishing value. Motivations from *M-theory* (A theory in physics that unifies all consistent versions of super string theory) predict that scalar field couplings with the Gauss-Bonnet invariant, G, are important in the appearance of non-singular early time cosmologies, in other words, *f(G)* gravity represents a interesting gravitational alternative to explain the accelerated expansion of the universe. There is the possibility to describe the in inflationary Era, a transition from a deceleration phase to a acceleration phase.

One of the main goals of modern cosmology is the looking for a answer to the large scale structure formation, the current distribution of mass and energy is due to a small in homogeneities were present in the very earliest times of the cosmos, and due to their gravitational attraction, these fluctuations in the matter grew, eventually condensing into the galaxies and other structures that we see today. What was the origin of the initial perturbations that eventually grew? In try to explain this density in homogeneities in the current universe, we adopted the model of the origin of large scale structure, the cosmic string model. Cosmic strings are topologically stable gravitational defects which appear in the framework of grand unified theories. These objects could be produced in very early Universe as a result of spontaneous breakdown of gauge symmetry (Vilenkin and Shellard, 1994).

If they exist, the cosmic strings, they may help to explain some of the largest-scale structures seen in the Universe today, they are topological defects, that may have been formed at phase transitions in the very early history of the Universe, analogous to those found in some condensed-matter systems, vortex lines in liquid helium, flux tubes in *type II* superconductors, or disclination lines in liquid crystals. The strings are enormously heavy, with a mass per unit length of order $10^{19} kg/cm$. These numbers are obtained under the assumption that cosmic strings are indeed responsible for the primordial density perturbations, by fitting to the cosmic microwave observations (Vilenkin and Shellard, 1994). In this picture, the perturbations arose from a network of linear defects that were created during a phase transition that took place during the first 10^{-37} seconds after the big bang. The topic of cosmic strings provides a bridge between the physics of the very small and the very large.

The cosmic strings, main object of our study, are an important source of study in gravitation and cosmology branch, with the goal to better understand the current Universe, jointly with the Gauss-Bonnet gravity, an important theory of the gravitation interaction, we'll get the spacetime of a cosmic string in Gauss-Bonnet Gravity, emphasizing the difference between the General Relativity and Gauss-Bonnet Gravity model.

2. Field equations

Immediately after Einstein propose the General Theory of Relativity and Hilbert has found the action to describe it, Kretschmann showed that the general covariance wasn't enough to explain the action form (E.Kretschmann, 1917). Kretschmann introduced another kind of scalar, the Kretschmann's scalar,

$$S = \int d^4\, x\sqrt{-gR_{\alpha\beta\gamma\delta}}R^{\alpha\beta\gamma\delta}, \tag{1}$$

This action form had a good justification, the Riemann tensor is a fundamental tensor of gravitation, and the scalar form $R_{\alpha\beta\gamma\delta}\, R^{\alpha\beta\gamma\delta}$ can be built. Moreover, This is a theory in the scenario which the Bianchi's identities remain valid, both sides are preserved.

To avoid higher order terms in the equations, we'll consider the following Gauss-Bonnet scalar

$$G = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \tag{2}$$

If the Gauss-Bonnet scalar is used in a n-dimensional action,

$$S = \int d^4\, x\sqrt{-gG}, \tag{3}$$

The equations produced are just of the second order derivative. To include mass and energy in our system, and to generalize the Gauss-Bonnet scalar, we rewrite (3) by adopting $\kappa = 8\pi G = 1$:

$$S = \int d^4\, x\sqrt{-g\left[f\,(G) + \frac{R}{2}\right] + L}. \tag{4}$$

L is the Lagrange density. Deriving (4) with respect to the metric tensor $g_{\mu\nu}$, we found the following equations:

$$G_{\mu\nu} + 8\big[R_{\mu\rho\nu\sigma} + R_{\rho\nu}g_{\sigma\mu} + {}^{1}\!/_{2}\,(g_{\mu\nu}g_{\sigma\rho} - g_{\mu\nu}g_{\sigma\rho})R - R\rho\sigma g\nu\mu - R\rho\sigma g\nu\mu - R\mu\nu g\sigma\rho + \tag{5}$$

$$g_{\nu\rho}\big]\nabla^{\rho}\nabla^{\sigma}F(G) + g_{\mu\nu}[GF(G) - f(G)] = kT_{\mu\nu} \tag{6}$$

$G_{\mu\nu}$ being the Einstein tensor, $R_{\mu\nu\,\alpha\beta}$ the Riemman tensor, R the Ricci scalar, $T_{\mu\nu}$ the energy momentum tensor and $F\,(G) = df(G)/dG$. If we make $f(G) = G$, in this last equation, we obtain the standard Einstein solution. In this model, Eq. (6), can easily unify the early inflation with late-time cosmic acceleration for the special choice of gravitational functions. The non-minimal coupling G and matter lagrangian, can be reduced from higher dimensional theories such as brane theory and can get rid of the Big Bang Singularity or lead to the cosmic acceleration.

3. The Cosmic String in modified gravity

Topological defects are structures formed in a field theory from a spontaneous symmetry breaking

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belonging to a physical system that presents a degenerate set of vacuum states. Each type of broken symmetry there is an associate defect type. In the cosmological scenario this means that the early universe passed through a series of phase transitions, where the associates symmetries were spontaneously broken due to natural cooling, building the topological defects, among them we can mention: Monopoles, Domain walls and Cosmic Strings, the monopoles would have annihilated with the opposites, called *anti-monopole*, and the domain walls annihilated themselves making cosmic radiation, remain only the cosmic string. Cosmic strings are one-dimensional topological defects, an important source of search in gravitation and cosmology that had their origin still in the early universe. The simplest theoretical model describing an idealized cosmic string, i.e., straight and infinitely thin, is given by a delta-type distribution for the energy-momentum tensor along the linear defect. As the solution of the Einstein equation, the geomentry of the spacetime produced by this source presents a conical singularity for the curvature tensor on its top. Under classical field theory viewpoint, this object can also be formed coupling the energy-momentum tensor associated with Higgs U (1)-gauge system investigated by Nielsen and Olesen (Nielsen & Olesen, 1973) with the Einstein equations. This project was successfully analysed by Garfinle a few years ago (D. Garfinkle, 1985). He found static cylindrically symmetric solutions representing vortices, as in at spacetime, and shown that asymptotically the space-time around the vortices is a Minkowski one minus a wedge., Their core have a non-zero thickness, and the magnetic fields vanishes outside them. Two years later Linet (B. Linet, 1987) obtained, as a limit case, exact solutions for the metric tensor and Higgs field. He was able to show that the structure of the respective space-time corresponds to a conical one, with the conicity parameter being expressed in terms of the energy per unity length of the vortex. The gravitational properties of strings are studied in the linear approximation in General Relativity frame by Vilenkin (Alexander Vilenkin, 1981), but was Hiscock who firstly got the exact solution for a static cylindrically symmetric string (W.A.Hiscock, 1985). Continuing the investigation about cosmic string and jointly with the modified gravity, we are ready to find a solution for the cosmic string in the Gauss-Bonnet scenario.

The $f(G)$ gravity, insert a scalar, more general than Ricci scalar, the generated field equations are different of the generated in the standard Einstein's gravity, the GB scalar is more refined than Ricci scalar, and consequently more embracing (De Felice *et al*, 2009).

To this specific spacetime, cosmic string in Gauss-Bonnet gravity, we have to make some restrictions to the internal spacetime of string: we consider a static and homogeneous string, i.e., there is no rotating and mass, the energy density will be evenly distributed, is located along z-axis, under these conditions, we write the line element in cylindrical coordinates:

$$ds^2 = A(r)dt^2 - dz^2 - dr^2 + C(r)^2 d\varphi^2. \tag{7}$$

$A(r)$ and $C(r)$ are functions of radial coordinate only. The energy momentum tensor is,

$$T_{\mu}^{\ \nu} = \sigma(r) diag(1.0,0,1), \tag{8}$$

Being $\sigma(r)$ the string energy density, given by:

$$\sigma(r) = \begin{cases} \sigma_{o,} & se \quad r \le r_{o,} \\ o, & se \quad r > r_{o,} \end{cases} \tag{9}$$

r_0 is the radius of string.

Making $\mu = \nu = 0$ in (5), we obtain the following results:

$$G_{00} + 8\left[R_{0\rho 0\sigma} + R_{\rho 0} g_{\sigma 0} + {}^1\!/_2 \left(g_{00} g_{\sigma\rho} - g_{0\sigma} g_{0\rho}\right) R - R\rho\sigma g00 - R00 g\sigma\rho + R0\sigma g0\rho \nabla^{\rho}\nabla^{\sigma} FG + g00GFG - f(G) = k2\ T00, \tag{10}$$

Where,

$$G_{00} = -\frac{A(r)}{C(r}\left(\frac{d^2A}{dr^2} + \frac{dC}{dr}\frac{dA}{dr} + \frac{d^2C}{dr^2}A\right) \tag{11}$$

$$R = \frac{2}{A^2C}\left[2AC\frac{d^2A}{dr^2} + 2A\frac{dA}{dr}\frac{dC}{dr} + \left(\frac{dA}{dr}\right)^2 C + \frac{d^2C}{dr^2}A^2\right], \tag{12}$$

$$R_{00} = \frac{1}{C}\left[AC\frac{d^2A}{dr^2} + AC\frac{dA}{dr} + \left(\frac{dA}{dr}\right)^2 C\right], \tag{13}$$

And the energy momentum tensor is,

$$T_{00} = \sigma(r)A(r)^2. \tag{14}$$

As demonstrated in the appendix, $A(r)$ is constant, this result is valid for any metric written in the form of (7) with a stress-tensor of the form (8). $A(r)$ being a constant, we can make for convenience and make the simplest equations, $A(r) = 1$, the equations between (11) to (14) becomes:

$$G_{00} = -\frac{1}{C(r)}\frac{d^2C}{dr^2}, \tag{15}$$

$$R = \frac{1}{C(r)}\frac{d^2C}{dr^2}, \tag{16}$$

$$R_{00} = 0, \tag{17}$$

$$T_{00} = \sigma(r), \tag{18}$$

The Riemann tensor are given by: $R_{0101} = R_{0202} = R_{0303} = 0$. The Gauss-Bonnet invariant, G, is calculated from (2), namely:

$$G = -3\left(\frac{1}{C}\frac{d^2C}{dr^2}\right)^2. \tag{19}$$

$C(r)$ is a function of r-coordinate only. Replacing whole equations from (11) to (14) jointly with,

$$R_{22} = -C\frac{d^2C}{dr^2} \tag{20}$$

In (10), we obtain:

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$$[GF(G) - f(G)] - \frac{1}{C}\frac{d^2C}{dr^2} = \sigma(r) \qquad (21)$$

where $f(G)$ is a Gauss-Bonnet function not yet defined, $F(G)$ represents the derivative of $f(G)$ with respect to GB scalar, G, e $\sigma(r)$ is the energy density of the string, written in (9).

The function *f (G)* can be written in many forms, there are many possibilities and many considerations that must be taken into account for their final choice, we preferred the Cognola's choice (Banijamali & Fazlpour, 2012). Between many motivations for the choice, we can cite few: simplification of the field equations, possibility to use the energy conditions (if necessary), models of this kind can produced quintessence, ghosts or cosmological constant effects; explains the universe's acceleration without dark energy; can use this model to explain phase transition in the early universe (Banijamali & Fazlpour, 2012). Thus, the Cognola's choice for the Gauss-Bonnet function is:

$$f(G) = \alpha G^n, \qquad (22)$$

α is a constant and n a integer not yet defined. The first derivative of (22) is:

$$f(G) = n\alpha G^{n-1}. \qquad (23)$$

Replacing (19), (22) and (23), the eq. (21) take the following form:

$$\propto (n-1)\left[-3\frac{1}{C(r)^2}\left(\frac{d^2C}{dr^2}\right)^2\right]n - \frac{1}{C(r)}\frac{d^2C}{dr^2}\sigma(r). \qquad (24)$$

Due to the impossibility to solve this equation, (24), in the general form, we'll make some choice. To $n = 0$ or $n = 1$, we obtained the usual solution to the cosmic string in Einsteins gravity. The next choice is obviously $n = 2$. Whatever the choice for n, we have two distinct situations to analyze, the internal and external part of the string, both differentiated by the energy density:

$$\sigma(r) = \begin{cases} \sigma_{o,} & se \quad r \leq r_{o,} \\ o, & se \quad r > r_{o,} \end{cases} \qquad (25)$$

Let's start with the region $r < r_0$.

3.1. Internal Solution

In this situation, the energy density is constant for the internal region, the same is true for mass density. Therefore, from eq. (25) we have $\sigma(r) = \sigma_0$ and the equation (24) to $n = 2$ become:

$$9\alpha\left[\frac{1}{C(r)}\left(\frac{d^2C}{dr^2}\right)\right]^4 - \frac{1}{C(r)}\frac{d^2C}{dr^2} = \sigma_{0.} \qquad (26)$$

The mathematical relation between α and σ_0 is very important to solve this equation, α is a parameter that we can adjust, has no restrictions to α. To some σ_0 values, the equation (26) has no solution, therefore, the parameter α and the energy density σ_0, must be chosen to allow that the equation (26) have a solution. In general way, this equation will only solution to positive values of the σ_0.

In order to find a solution to equation (26) we'll make the following choose:

$$\varepsilon^2 = \frac{1}{C(r)}\left(\frac{d^2C}{dr^2}\right), \qquad (27)$$

ε^2 is a constant, what actually is true, if α is a constant too, since σ is a positive constant in the inside of the string. The term (27) must be adjusted to allow the expression (26) be a identity. The energy conditions imposed on the form of the funtion chosen (22), forces us to choose positive values to α parameter (Banijamali & Fazlpour, 2012).

Making $\zeta = \alpha^{1/3}\sigma_0$ and $x = \sigma^{1/3}\zeta$, the differential equation (26) becomes dimensionless,

$$\zeta(\sigma) = 9x(\alpha, r)^4 - x(\sigma, r). \qquad (28)$$

In this situation, only existing solution to (26) in the cases in that the energy density is positive. To $\chi \leq 0$ and $\chi \geq 3^{-2/3}$, ζ and consequently σ_0, are positives, see figure (1). Therefore, $\xi \geq 9\alpha^{-1/3}$ and $\xi \leq 0$, we have the energy conditions to $f(G)$ function satisfied. However, ξ can be a positive or negative factor, we must take this fact in account, to positive values in ξ factor, the cosmic string has a planar angular superavit, we do not wish, the spacetime of the cosmic string in others solutions presents a planar angular deficit, like in General Relativity. Thus, we can delete the positive values to ξ, the equation (27) becomes,

$$\left(\frac{d^2C}{dr^2}\right) + C(r)\varepsilon^2 = 0, \qquad (29)$$

The solution is given by,

$$C(r) = c_1\sin(\varepsilon r) + c_2 cos(\varepsilon r). \qquad (30)$$

Using the boundary condition,

$$C(r)\Big|_{r=0} = 0, \frac{dC(r)}{dr}\Big|_{r=0} = 1 \qquad (31)$$

We found that,

$$C(r) = \frac{1}{\varepsilon}\sin(\varepsilon r). \qquad (32)$$

Thus, the internal solution to the cosmic string in Gauss-Bonnet Gravity is written in the following form,

$$ds^2_{(\_)} = dt^2 - dr^2 - \frac{1}{\varepsilon^2}sin^2(\varepsilon r)d\emptyset^2 - dz^2. \qquad (33)$$

This solution is similar to the Hiscock's solution (W. A. Hiscock, 1985). Herein, we have a different situation; our angular deficit is dependent of the energy density s_0 and the choice of α parameter. If we choose different values to α, we would have a superavit angular, case not foreseen by the General Relativity.

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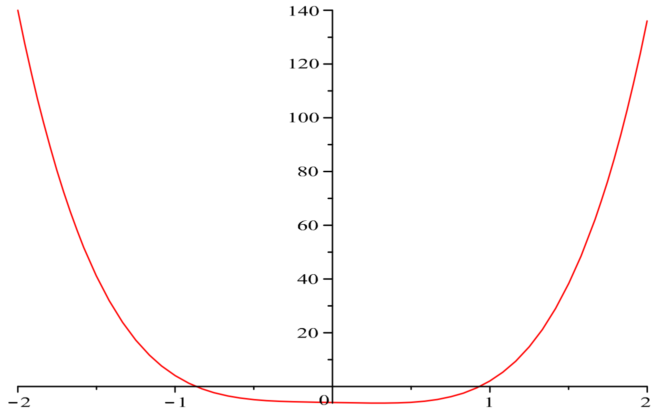

Figure 1: The figure shows the ζ -variation as a function of the energy density.

3.2. External Solution

The external solution to the cosmic string was firstly found for Hiscock in 1985, describing an one-dimensional, infinity and axially symmetric cosmic string (W.A.Hiscock, 1985). In order to find an external solution, $r > r_0$, in modified gravity $F\,(G)$, we'll follow similar steps made in the Hiscock's solution. We consider as a general solution for an object axially symmetric the Levi-Civita metric (T. Levi-Civita, 1917):

$$ds^2 = r^{2m}dt^2 - r^{-2m}\left[r^{2m^2}(dr^2 + dz^2) + a^2r^2d\emptyset^2\right] \quad (34)$$

Being a and m both constants. In order to find a solution to our specific spacetime and to obey the energy conditions imposed on cosmic string, we make the follow choose, $m = 0$, the metric (34) becomes,

$$ds^2 = dt^2 - dr^2 - a^2e^2d\emptyset^2 - dz^2. \quad (35)$$

We imposed that the cosmic string spacetime is continuous in their boundary, i.e., $r = r_0$, r_0 being the cosmic string radius, thus, the metric and our first derivative are physically continuous in the string's boundary, i.e., $g^{+}_{\mu\nu}$ and $g^{-}_{\mu\nu}$ are the external and internal metric respectively.

$$g^{(-)}_{22}|_{r=r_0} = g^{+}_{22}|_{r=r_0} \frac{dg^{-}_{22}}{dr}\Big|_{r=r_0} \frac{dg^{+}_{22}}{dr}\Big|_{r=r_0} \quad (36)$$

Using this relation we found the following result,

$$a = cos\,(\varepsilon r_0). \quad (37)$$

The string superficial mass density is given by:

$$\mu = \sigma_0 \int \sqrt{-g^{(-)}d\sigma} \quad (38)$$

$g^{(-)}$ being the internal determinant of the string spacetime and $d\sigma$ a two-dimensional surface given by $d\sigma = d\varphi dr$. Integrating (38) and replacing (37), we found:

$$a = \left(1 - \frac{\varepsilon^2\mu}{2\pi\sigma_0}\right). \quad (39)$$

Finally, the solution for a cosmic string in modified gravity $f(G)$ is written as,

$$ds^2 = dt^2 - dr^2 - \left(1 - \frac{\varepsilon^2\mu}{2\pi\sigma_0}\right)^2 r^2d\emptyset^2 - dz^2. \quad (40)$$

One more time, the angular deficit is different from that found in Hiscock's solution (W.A.Hiscock, 1985), in both cases, internal and external solution. Our solution for the external part, (40), is dependent of the term ξ, given in (28), forced us to specify values for him, that showed us that the solution in Gauss-Bonnet Gravity for the string it seems be related with the observer position, the angular deficit is dependent of our position. This situation is strange, it is not foreseen by General Relativity, The physics of the cosmic string scenario for structure formation is complicated, it is difficult to extract precise predictions that can be compared with the observations of the astronomers. We must go on studying the cosmic string space-time in others theories to better understand this situation.

4. Appendix

With the use of the energy conservation law we can demonstrate, to any metric written in the form of (7) and having a energy momentum tensor like (8) the function $A(r)$ is ever ∂ constant.

The covariant derivative of the energy momentum tensor must be null:

$$\nabla_\mu T^{\mu}_{\ \nu} = 0 \quad (41)$$

So that,

$$\partial_\mu T^{\mu}_{\ \nu} + \Gamma^{\mu}_{\mu\alpha}T^{\sigma}_{\ \nu} - \Gamma^{\mu\sigma}_{\mu\nu}T^{\mu}_{\ \sigma} = 0 \quad (42)$$

The only non-zero component is

$$-\Gamma^{0}_{01}T^{0}_{\ 0} = 0 \quad (43)$$

Thus,

$$\Gamma^{0}_{01} = 0 \quad (44)$$

Using the expression Γ^{0}_{01}, and replacing in (44), we obtained the following result

$$\frac{dA(r)}{dr} = 0, \quad (45)$$

In others words, $A(r)$ is a constant.

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