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Cost Analysis of a Two Dissimilar-Unit Cold Standby System with Preventive Maintenance by Kolmogorov's Forward Method

M.Y Haggag[1]* & Ahmed Khayar[2]

[1]&[2]Department of Mathematics
Faculty of Science
Al-Azhar University
Nasr City 11884, Cairo, Egypt.

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ABSTRACT

Many authors have studied the cost analysis of a two-similar unit cold standby redundant system with two types of unit failure, but no attention was paid to the reliability of a system with two dissimilar units and preventive maintenance. Question was raised whether the preventive maintenance would be effective on the reliability and performance of the system. The paper introduces a statistical analysis of a two-dissimilar unit cold standby redundant system with two types of failures and preventive maintenance to evaluate availability, MTSF and cost analysis and determine the efficacy of preventive maintenance on the reliability and performance of the system. In the study undertaken, the mean time to system failure (MTSF), steady state availability and the profit function of a two- unit cold standby repairable redundant system involving preventive maintenance were discussed. The system was analyzed by using Kolmogorov's forward equations method. Some particular cases have also been discussed graphically. Some particular cases studies the effect of preventive maintenance on the system performance are shown by tables and graphs. The results indicated that the system with preventive maintenance is better than the system without preventive maintenance. These results indicated that the better maintenance of parts of the system originated better reliability and performance of the system.

Keywords: Reliability, Mean Time to System Failure (MTSF), steady-state availability, busy period, profit function, Failure and repair time, cost analysis, Expected frequency, preventive maintenance, Kolmogorov's Forward Equations Method.

1- Introduction

Various reliability systems often come to maximize the profit. Better maintenance of a system originates better reliability, and performance of a system. Also, in a standby redundant system, some additional paths are created for the proper functioning of the system. The standby unit support increases the reliability of the system. On the failure of the operating unit, a standby unit is switched on by perfect or imperfect switching device. Thus introducing redundant parts and providing maintenance and repair may achieve high degree of reliability.

Earlier researchers have studied cost analysis of a system with preventive maintenance. El-saied, 2008 have studied cost analysis of a system with preventive maintenance by using the Kolmogorov's forward equations method. Goel, et al 1986a have studied reliability analysis of a system with preventive maintenance and two type of repair. Goel, et al 1986b have studied Profit analysis of a cold standby system with two repair distribution. Elias,&haggag1991 have studied Cost analysis of one-server two-dissimilar unit repairable parallel redundant system with two states under human failure. Other researchers have studied the cost analysis of different systems. Gopalan,&Nagarwalia, 1985 have studied Cost benefits analysis of a one server two-unit cold stand by system with repair and preventive maintenance. Goel, et al 1985 have studied Cost analysis of a two-unit cold standby system with two types of operation and repair. El-Said,&El-Sherbeny 2005 have studied Profit analysis of a two unit cold standby system with preventive maintenance and random change in units. Wang, et al 2006. Have studied Cost benefit analysis of series systems with cold standby components and a repairable service station. Randeret, et al 1994 have studded cost analysis of two dissimilar cold standby systems with preventive maintenance and replacement of standby. Haggag. 2009 has studied Cost Analysis of Two-Dissimilar-Unit Cold Standby System with three States and Preventive Maintenance using Linear First Order Differential Equations. Journal of Mathematics and Statistics 5(4) :395-400. Haggag and Khayar 2010 have studied Cost Analysis of a Two-Unit Cold Standby Redundant System with General Repair Rates and Preventive Maintenance.

The purpose of this paper is to study the cost analysis of a two-dissimilar unit cold standby redundant system with two types of unit failure and preventive maintenance. The failure and repair rate times follow exponential distribution Several reliability characteristics are obtained. The system was analyzed by using Kolmogorov's forward equations method. Initially one unit is operative and the other is kept as cold standby. Each unit works in two different types of failures. The systems fail when both units fail totally. The failure and repair times are assumed to have exponential distribution. The availability, steady state availability, mean time to system failure (MTSF) and cost function are evaluated. Some particular cases study the effect of preventive maintenance on the system performance is shown by tables and graphs.

Corresponding Author: M.Y. Haggag[1]*
Faculty of Science, Department of Mathematics, Al-Azhar University, Cairo, Egypt.
Email address: mohyas62@yahoo.com

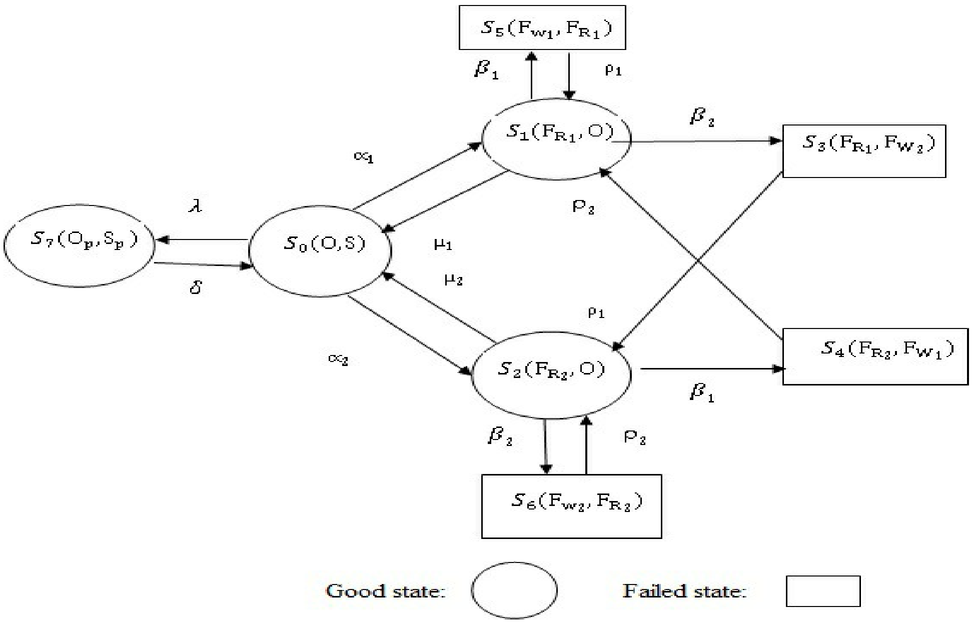

Figure (1) State of the system

The following system characteristics are obtained

1. Mean time to system failure (MTSF) with and without preventive maintenance.
2. Steady state availability with and without preventive maintenance.
3. Cost analysis with and without preventive maintenance.

2- Assumptions:

The following assumptions were adhered to in this work:

4. The system consists of two-dissimilar units, one is main and the other is its standby.
5. Initially one unit is operative and the other unit is kept as cold standby.
6. A perfect switch is used to switch-on standby unit and switch-over time is negligible.
7. The system has three states: good, failed, and under preventive maintenance.
8. Both units suffer two types of failures.
9. Unit failure and repair rates are constants.
10. Failure and repair rates follow an exponential distribution.
11. A repaired unit works as good as new.
12. The system is down when both units are non-operative.
13. The system can reach a failed states S_3, S_4, S_5, S_6 due to unit failure for its two units. (see Figure 1).

Notations

α_1 : constant failure rate of type I for main unit

α_2 : constant failure rate of type II for main unit

β_1 : constant failure rate of type I for standby unit

β_2 : constant failure rate of type II for standby unit

μ_1 : constant repair rate of type I for main unit

μ_2 : constant repair rate of type II for main unit

ρ_1 : constant repair rate of type I for standby unit

ρ_2 : constant repair rate of type II for standby unit

λ : constant rate for taking a unit into preventive maintenance.

δ : constant rate end of preventive maintenance

O : the unit is operative.

S : the unit is standby.

F_{R1} : the failed unit is under repair of type I.

F_{R2} : the failed unit is under repair of type II.

F_{W1} : the failed unit is waited for repair of type I.

F_{W2} : the failed unit is waited for repair of type II.

O_p : the operative unit is under preventive maintenance.

S_p : the standby unit is under preventive maintenance.

$P_i(t)$: Probability that the system is in state S_i at time t, , $(t \geq 0)$, i = [0- 7].

3- Reliability Assessment

The mean time to system failure (MTSF) for the proposed system was evaluated using the above-mentioned set of assumptions and method of linear first order differential equations. Let P(t) denote the probability row vector at time t,, the initial conditions for this problem are as presented in equation 1:

$$P(0) = [P_0(0)\ \ P_1(0)\ \ P_2(0)\ \ P_3(0)\ \ P_4(0)\ \ P_5(0)\ \ P_6(0)\ \ P_7(0)\ \ P_8(0)] = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \qquad (1)$$

By employing the method of linear first order differential equations and for Fig. 1, the set of differential equations (2) can be obtained.

$$\left.\begin{aligned}
P'_0(t) &= -(\alpha_1+\alpha_2+\lambda)P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) + \delta P_7(t)\\
P'_1(t) &= -(\beta_1+\beta_2+\mu_1)P_1(t) + \alpha_1 P_0(t) + \beta_1 P_4(t) + \beta_1 P_5(t)\\
P'_2(t) &= -(\beta_1+\beta_2+\mu_2)P_2(t) + \alpha_2 P_0(t) + \beta_2 P_3(t) + \beta_2 P_6(t)\\
P'_3(t) &= -\rho_2 P_3(t) + \beta_2 P_1(t)\\
P'_4(t) &= -\rho_1 P_4(t) + \beta_1 P_2(t)\\
P'_5(t) &= -\rho_1 P_5(t) + \beta_1 P_1(t)\\
P'_6(t) &= -\rho_2 P_6(t) + \beta_2 P_2(t)\\
P'_7(t) &= -\delta P_7(t) + \lambda P_0(t)
\end{aligned}\right\} \qquad (2)$$

The above system of differential equations (2) can be written in the matrix form as depicted in equations 3 and 4.

$$P^* = Q \times P \qquad (3)$$

Where,

$$Q = \begin{bmatrix}
-(\alpha_1+\alpha_2+\lambda) & \mu_1 & \mu_2 & 0 & 0 & 0 & 0 & \delta\\
\alpha_1 & -(\beta_1+\beta_2+\mu_1) & 0 & 0 & \rho_1 & \rho_1 & 0 & 0\\
\alpha_2 & 0 & -(\beta_1+\beta_2+\mu_2) & \rho_2 & 0 & 0 & \rho_2 & 0\\
0 & \beta_2 & 0 & -\rho_2 & 0 & 0 & 0 & 0\\
0 & 0 & \beta_1 & 0 & -\rho_1 & 0 & 0 & 0\\
0 & \beta_1 & 0 & 0 & 0 & -\rho_1 & 0 & 0\\
0 & 0 & \beta_2 & 0 & 0 & 0 & -\rho_2 & 0\\
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta
\end{bmatrix} \qquad (4)$$

Mean Time to System Failure (MTSF)

To calculate the MTSF the transpose matrix of Q is taken and rows and columns are deleted for the absorbing state. The new matrix is called A the expected time to reach an absorbing state is calculated from equation 5.

$$MTSF = P(0)(-A^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \qquad (5)$$

Where,

$$A = \begin{bmatrix}
-(\alpha_1+\alpha_2+\lambda) & \alpha_1 & \alpha_2 & \lambda\\
\mu_1 & -(\beta_1+\beta_2+\mu_1) & 0 & 0\\
\mu_2 & 0 & -(\beta_1+\beta_2+\mu_2) & 0\\
\delta & 0 & 0 & -\delta
\end{bmatrix}$$

The steady state mean Time to System Failure (MTSF) is given by equation 6.

$$MTSF = (\lambda[(\beta_1+\beta_2+\mu_1)(\beta_1+\beta_2+\mu_2)] + \delta[(\beta_1+\beta_2+\mu_1)(\beta_1+\beta_2+\mu_2) + \alpha_1(\beta_1+\beta_2+\mu_2) + \delta\alpha_2(\beta_1+\beta_2+\mu_1)])/\{\delta((\alpha_1+\alpha_2)(\beta_1+\beta_2)^2 + \alpha_1\mu_2(\alpha_1+\alpha_2) + \alpha_2\mu_1(\alpha_1+\alpha_2))\} \qquad (6)$$

4- Availability analysis

The initial conditions for this problem are the same as for the reliability case:

P(0) = [1, 0, 0, 0, 0, 0, 0, 0], the differential equations form can be expressed as shown in equation 7.

$$\begin{bmatrix} P_0^{*} \\ P_1^{*} \\ P_2^{*} \\ P_3^{*} \\ P_4^{*} \\ P_5^{*} \\ P_6^{*} \\ P_7^{*} \end{bmatrix} = \begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \mu_1 & \mu_2 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\beta_1+\beta_2+\mu_1) & 0 & 0 & \rho_1 & \rho_1 & 0 & 0 \\ \alpha_2 & 0 & -(\beta_1+\beta_2+\mu_2) & \rho_2 & 0 & 0 & \rho_2 & 0 \\ 0 & \beta_2 & 0 & -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & -\rho_1 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -\rho_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\rho_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix}$$

(7)

In the steady state, the derivatives of the state probabilities become zero as presented in equation 8.

$$QP(\infty) = 0 \tag{8}$$

Then the steady state probabilities can be calculated as revealed in equation 9.

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty) \tag{9}$$

Then the matrix form became as displayed in equation 10.

$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \mu_1 & \mu_2 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\beta_1+\beta_2+\mu_1) & 0 & 0 & \rho_1 & \rho_1 & 0 & 0 \\ \alpha_2 & 0 & -(\beta_1+\beta_2+\mu_2) & \rho_2 & 0 & 0 & \rho_2 & 0 \\ 0 & \beta_2 & 0 & -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & -\rho_1 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -\rho_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\rho_2 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

To obtain:

$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty)$,

equation (8) must be solved by using the normalizing condition indicated in equation 11.

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) = 1 \tag{11}$$

Substituting the equation (11) in any one of the redundant rows in equation (8) yields equation 12.

$$\begin{bmatrix} -(\alpha_1+\alpha_2+\lambda) & \mu_1 & \mu_2 & 0 & 0 & 0 & 0 & \delta \\ \alpha_1 & -(\beta_1+\beta_2+\mu_1) & 0 & 0 & \rho_1 & \rho_1 & 0 & 0 \\ \alpha_2 & 0 & -(\beta_1+\beta_2+\mu_2) & \rho_2 & 0 & 0 & \rho_2 & 0 \\ 0 & \beta_2 & 0 & -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 & -\rho_1 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -\rho_1 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\rho_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{12}$$

The steady state availability A (∞) is given by equation 13.

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty) = (\rho_1\rho_2(\lambda(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2) + \delta(\alpha_1(\beta_1 + \beta_2 + \mu_2) + \alpha_2(\beta_1 + \beta_2 + \mu_1) + \beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)))/D \tag{13}$$

Where,

$$D = (\lambda\rho_1\rho_2(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2) + \delta(\rho_1 (\beta_1\beta_2(\alpha_1 + \alpha_2) + \beta_2^2(\alpha_1 + \alpha_2)) + \rho_2(\beta_1\beta_2(\alpha_1 + \alpha_2) + \beta_1^2(\alpha_1 + \alpha_2)) + \rho_1\rho_2(\alpha_1(\beta_1 + \beta_2 + \mu_2) + \alpha_2(\beta_1 + \beta_2 + \mu_1) + \beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2) + (\beta_1\rho_2 + \beta_2\rho_1)(\alpha_1\mu_2 + \alpha_2\mu_1)))) \quad (14)$$

Busy period analysis: The initial conditions for this problem are the same as for the reliability case: the differential equations form can be expressed as availability case. Then the steady state busy period B is given by equation 15.

$$B(\infty) = 1 - \left(P_0(\infty) + P_7(\infty)\right) = (1 - (\rho_2\rho_1(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)(\lambda + \delta))/D) \quad (15)$$

The expected frequency of preventive maintenance:

The initial conditions for this problem are the same as for the reliability case. Then the steady state, the expected frequency of preventive maintenance per unit time K is given by equation 16.

$$K(\infty) = P_7(\infty) = ((\rho_2\rho_1\lambda(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2))/D) \quad (16)$$

Cost analysis: The expected total profit per unit time incurred to the system in the steady-state is given by equation 17.

$$\text{Profit} = \text{total revenue - total cost} \quad (17a)$$
$$PF = RA(\infty) - C_1B(\infty) - C_2K(\infty) \quad (17b)$$

Where,

PF: is the profit incurred to the system,
R: is the revenue per unit up-time of the system,
C_1 : is the cost per unit time when the system is under repair
C_2: is the cost per preventive maintenance.

From equations (13), (15), (16), the expected total profit per unit time incurred to the system in the steady-state is given by equation 18.

$$PF = ((R(\rho_1\rho_2(\lambda(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2) + \delta(\alpha_1(\beta_1 + \beta_2 + \mu_2) + \alpha_2(\beta_1 + \beta_2 + \mu_1) + \beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2))) - C_1(D - \rho_2\rho_1(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)(\lambda + \delta)) - C_2(\rho_2\rho_1\lambda(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)))/D) \quad (18)$$

6- Special case

When the preventive maintenance is not available,

The Mean Time to System Failure is given by equation 19.

$$MTSF = ([(\beta_1 + \beta_2 + \mu_1)(\beta_1 + \beta_2 + \mu_2) + \alpha_1(\beta_1 + \beta_2 + \mu_2) + \delta\alpha_2(\beta_1 + \beta_2 + \mu_1)])/\{((\alpha_1 + \alpha_2)(\beta_1 + \beta_2)^2 + \alpha_1\mu_2(\alpha_1 + \alpha_2) + \alpha_2\mu_1(\alpha_1 + \alpha_2))\} \quad (19)$$

The steady state availability of the system is given by equation 20.

$$A(\infty) = (\rho_1\rho_2((\alpha_1(\beta_1 + \beta_2 + \mu_2) + \alpha_2(\beta_1 + \beta_2 + \mu_1) + \beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)))/D_1 \quad (20)$$

The steady state busy period of the system is given by equation 21.

$$B(\infty) = (1 - (\rho_2\rho_1(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)))/D_1) \quad (21)$$

The expected total profit incurred to the system in the steady-state is given by equation 22.

$$PF = R(\rho_1\rho_2((\alpha_1(\beta_1 + \beta_2 + \mu_2) + \alpha_2(\beta_1 + \beta_2 + \mu_1) + \beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2))) - C(D_1 - \rho_2\rho_1(\beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2)))//D_1 \quad (22)$$

Where,

$$D_1 = (\rho_1 (\beta_1\beta_2(\alpha_1 + \alpha_2) + \beta_2^2(\alpha_1 + \alpha_2)) + \rho_2(\beta_1\beta_2(\alpha_1 + \alpha_2) + \beta_1^2(\alpha_1 + \alpha_2)) + \rho_1\rho_2(\alpha_1(\beta_1 + \beta_2 + \mu_2) + \alpha_2(\beta_1 + \beta_2 + \mu_1) + \beta_1\mu_1 + \beta_2\mu_2 + \mu_1\mu_2) + (\beta_1\rho_2 + \beta_2\rho_1)(\alpha_1\mu_2 + \alpha_2\mu_1)))) \quad (23)$$

8- Numerical computation

Put α_2=,04,β_1=.05,β_2=.06,λ=.1,δ=.1,μ_1=.2,μ_2=.3,ρ_1=.4,ρ_2=.5 in equations (6), (13), (18) and equations (19) , (20) , (22) the following is revealed.

1- Table (1): Shows relation between failure rate of type I and the MTSF of the system (with and without PM)
2- Table (2): Shows relation between failure rate of type I and the steady-state availability of the system (with and without PM).
3- Table (3): Shows relation between failure rate of type I and the profit of the system (with and without PM).
4- Fig. 2: Shows relation between the failure rate of type I and the MTSF of the system (with and without PM).
5- Fig. 3: Shows relation between the failure rate of type I and the steady-state availability of the system (with and without PM).
6- Fig. 4: Shows relation between the failure rate of type I and the expected total profit of the system (with and without PM).

Table (1): Relation between failure rate of type I and the MTSF (with and without PM)

α_1	MTSF of the system without PM	MTSF of the system with PM
0.01	124.33	355.42
0.02	94.317	264.76
0.03	74.521	205.55
0.04	60.709	164.65
0.05	50.649	135.14
0.06	43.071	113.13
0.07	37.204	96.242
0.08	32.559	82.995
0.09	28.809	72.401
0.1	25.734	63.788

Table (2): Relation between failure rate of type I and availability (with and without PM)

α_1	Availability of the system without PM	Availability of the system with PM
0.01	0.96276	0.97943
0.02	0.95559	0.97497
0.03	0.94904	0.97073
0.04	0.94303	0.96669
0.05	0.9375	0.96285
0.06	0.93238	0.95918
0.07	0.92765	0.95567
0.08	0.92324	0.95232
0.09	0.91914	0.94912
0.1	0.91531	0.94605

Table (3): Relation between failure rate of type I and the profit (with and without PM)

α_1	The profit of the system without PM	The profit of the system with PM
0.01	943.83	1496. 1
0.02	933.02	1476
0.03	923.14	1456.9
0.04	914.08	1438.8
0.05	905.73	1421.5
0.06	898.02	1405
0.07	890.88	1389.2
0.08	884.24	1374.1
0.09	878.05	1359.7
0.1	872.28	1345.9

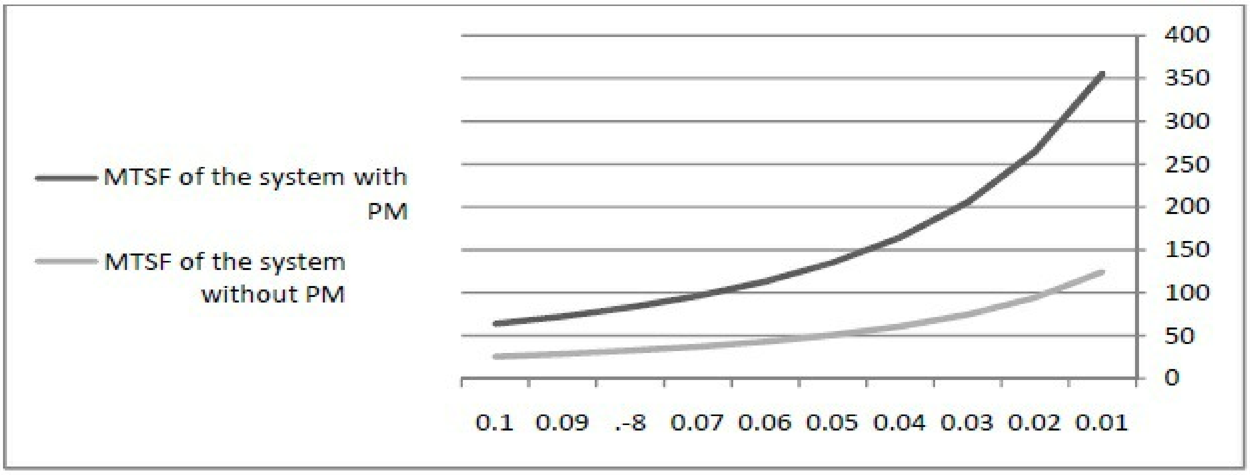

Fig. 2: Relation between the failure rate of type I and the MTSF

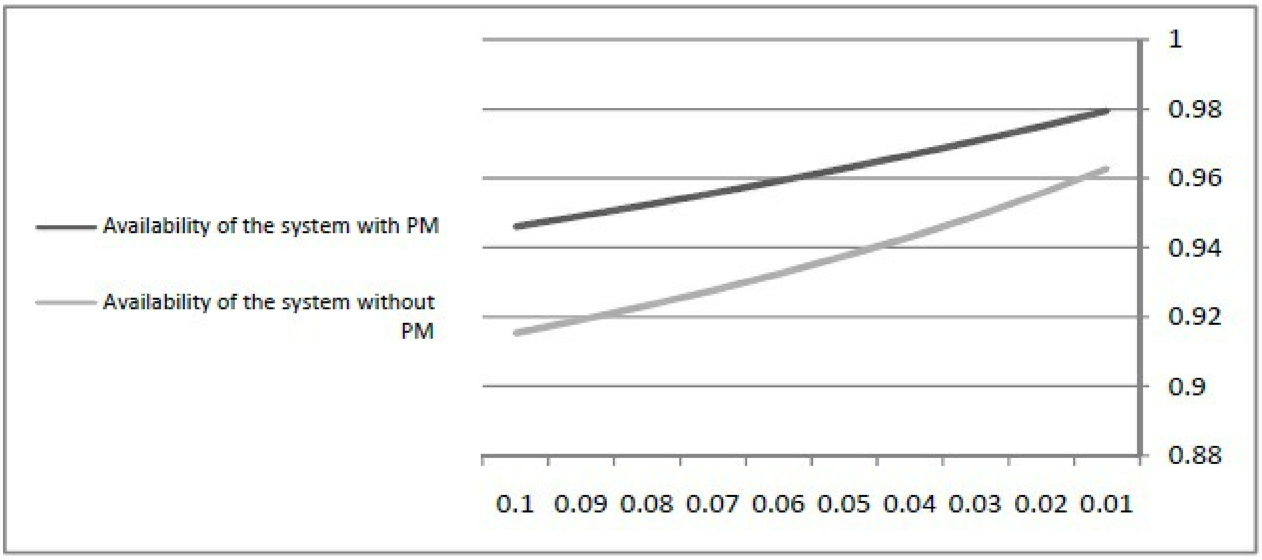

Fig. 3: Relation between the failure rate of type I and the Availability

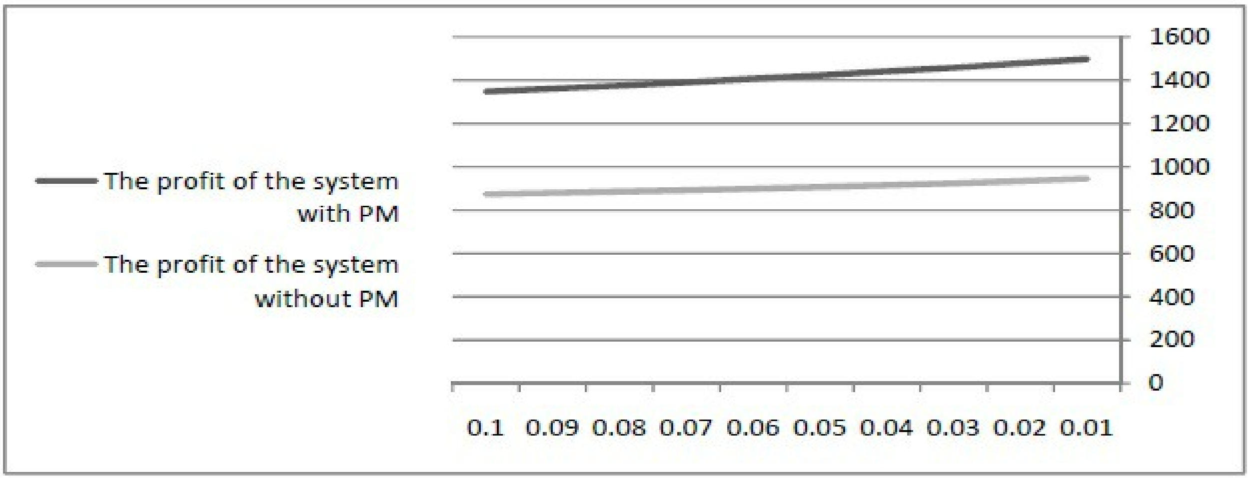

Fig. 4: Relation between the failure rate of type I and the expected total profit

Conclusion

By comparing the characteristic, MTSF, Availability and the profit function with respect to α_1 for both systems with and without preventive maintenance graphically, it was observed that there is an increase of failure rate α_1 at constant α_2=,04, β_1=.05, β_2=.06, λ=.02, δ=.02, γ=.001, η=.04, R =1000, C1= 100, C2 = 50. Likewise, the MTSF, availability and the profit function of the system decreased for both systems with and without preventive maintenance.

It was also observed that the system with preventive maintenance is more greater than the system without preventive maintenance with respect to the MTSF, steady-state availability and the profit function. It is concluded that the system with preventive maintenance is better than the system without preventive maintenance.

References

1. El-saied, K., M., .2008. Cost analysis of a system with preventive maintenance by using the Kolmogorov's forward equations method, American Journal of Applied sciences 5(4), pp. 405-410
2. Goel, L.R., Sharma, G.C. and Gupta, P., 1986a. Reliability analysis of a system with preventive maintenance inspection and two type of repair. Micro-electron. Reliab., 26: pp./ 429-433.
3. Goel, L.R.,.Gupta R.and Singh, S.K., 1986b. Profit analysis of a cold standby system with two repair distribution. Micro-electron. Reliab., 25: pp. 467-472.
4. Elias S. S. & Haggag M. Y., 1991. Cost analysis of one-server two-dissimilar unit repairable parallel redundant system with two states under human failure, Microelectronic Reliability.
5. Gopalan, M.N. and Nagarwalia, H.E., 1985. Cost benefits analysis of a one server two-unit cold stand by system with repair and preventive maintenance. Micro-electron. Relib., 25, pp. 267-269.
6. Goel L.R., Gapta R. and Singh S. K., 1985. Cost analysis of a two-unit cold standby system with two types of operation and repair, Microelectronic Relliab., Vol, 25, No 1 pp. 71-75.
7. El-Said, M. Kh. and El-Sherbeny, M.S., 2005. Profit analysis of a two unit cold standby system with preventive maintenance and random change in units. J. Math. Stat., 1, pp. 71-77.
8. Wang, K.H., Hsieh C.H. and Liou, C.H., 2006. Cost benefit analysis of series systems with cold standby components and a repairable service station. Qual. Technol. Quant. Manag., 3, pp. 77-92
9. Rander, M.C., Suresh K. and Ashok K., 1994. Cost analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby. Microelectron. Reliab., 34: pp. 171-174.
10. M. Y. Haggag. Cost Analysis of Two-Dissimilar-Unit Cold Standby System with three States and Preventive Maintenance using Linear First Order Differential Equations. Journal of Mathematics and Statistics 5(4) :395-400, 2009.
11. M. Y. Haggag and Ahmed Khayar. Cost Analysis of a Two-Unit Cold Standby Redundant System with General Repair Rates and Preventive Maintenance.Al-Azhar Bull.Sci. Vol. 21, No. 2 (Dec) pp. 1-8, 2010.

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